Math 308 **Midterm Practice** **Spring 2015**

The Midterm will take place Nov 6th, during lecture time. It will cover the material of Chapter 1, Chapter 2 and Chapter 3 (as far as we got into that chapter.)
You do not need to bring paper to write on, because there will be enough room on the test paper. You are allowd to brin a sheet of handwritten note and a copy of the 'table'.
Be reminded that anything you write on the midterm test paper must be your own work. If there is evidence that you are claiming credit for work that is not your own work during the test period, I need to give you a zero on the test and turn the evidence over to the Dean's Committee on Academic Conduct.
Study suggestions: To be successful on the test I highly recommend to catch up on the homework. After that repeat solving problems. Find out with which kind of problems you are still struggling. Study those over and over again, so that you develop a routine. If there are questions that you just cannot find an answer for, do not hestitate to visit me during office hours. I am more than happy to explain whatever is unclear.
Exam advise: Start with whatever problem seems easy to you. Having solved something right away calms down and gives a secure feeling. Do not waste too much time on a specific problem if you get stuck. Rather start with another problem and come back later, if there is time left. I appreciate very much, if your work is layed down in a logic order and in readable writing. If you need more space than is available on the paper, give clear instructions about where to find the rest of your work. Place a box around your final answer.
On the following pages you find review problems. You need to turn your answers in at latest by Wedndesday 11/4, in order to get points towards the Midterm. Just for turning in this practice test, you will get approximately 7 % of the possible points of the exam - independently of how well you do in the practice test.

1.)

(a)	Cross the right box for the statements about linear systems.		
	There are homogeneous systems with no solution.	☐ TRUE	☐ FALSE
	Every linear system with more variables than equations has at least one solution	☐ TRUE	☐ FALSE
	If a linear system of the form $A\mathbf{x} = \mathbf{b}$ with A an (n, m)-matrix and $\mathbf{b}$ a vector in $\mathbb{R}^n$ has asolution, then this solution can be written as a vector in $\mathbb{R}^m$.	☐ TRUE	☐ FALSE
	Let $\mathbf{u}_i$ be vectors in $\mathbb{R}^n$. Then the linear system $[\mathbf{u}_1\ \mathbf{u}_2\ \dots\ \mathbf{u}_n \vert \mathbf{u}_2]$ has $[0, 1, 0, \dots, 0]^t$ as a solution.	☐ TRUE	☐ FALSE
	Any homogeneous system with 5 variables and 3 equations has infinitely many solutions.	☐ TRUE	☐ FALSE
	If $A\mathbf{x} = \mathbf{b}$ has a solution and $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions, then $A\mathbf{x} = \mathbf{b}$ has also infinitely many solutions.	☐ TRUE	☐ FALSE
(b)	Cross the right box for the statements about linear independence and span.		
	Including the zero vector in $\mathbb{R}^n$ always gives a linearly dependent set of vectors in $\mathbb{R}^n$.	☐ TRUE	☐ FALSE
	$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\} \subseteq \mathbb{R}^4$ is always linearly dependent.	☐ TRUE	☐ FALSE
	$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\} \subseteq \mathbb{R}^4$ always spans $\mathbb{R}^4$.	☐ TRUE	☐ FALSE
	If $\mathbf{u}_1$ and $\mathbf{u}_2$ are linearly dependent, then there exists a scalar $c \in \mathbb{R}$ such that $\mathbf{u}_1 = c\mathbf{u}_2$.	☐ TRUE	☐ FALSE
	If $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans $\mathbb{R}^3$, then so does $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$, for a vector $\mathbf{u}_4 \in \mathbb{R}^3$.	☐ TRUE	☐ FALSE
(c)	Cross the right box for the statements about linear systems and linear independence.		
	If A is an (n, m)-matrix and $\mathbf{b}$ a vector in $\mathbb{R}^n$ and the columns of A are linearly independent, then the corresponding linear system cannot have free variables.	☐ TRUE	☐ FALSE
	A set with two different vectors is linearly independent.	☐ TRUE	☐ FALSE
	If $\mathbf{u}_1, \mathbf{u}_2$, and $\mathbf{u}_3$ are pairwise linearly independent, then $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is also linearly independent.	☐ TRUE	☐ FALSE
	A homogeneous system with 3 variables and 3 equations has exactly one solution.	☐ TRUE	☐ FALSE
	The trivial solution is always a solution to a linear system.	☐ TRUE	☐ FALSE

2. Consider the following system:

$$\begin{array}{rcrcrcrcr} 2x_1 & + & & & 3x_3 & - & 5x_4 & = & 4 \\ x_1 & + & x_2 & - & x_3 & + & x_4 & = & 8 \\ 3x_1 & + & x_2 & + & 2x_3 & - & 3x_4 & + & 10 \end{array}$$

(a) Write down the corresponding augmented matrix.

(b) Perform Gauss-Jordan algorithm.

(c) After Gauss-Jordan, what is the form of the matrix called?

(d) Identify the leading variables.

(e) Identify the free variables.

(f) Determine the solutions of the variables. Write the solution in vector form (i.e. $\mathbf{x} = \mathbf{v} + s\mathbf{w}$, where you need to specify $\mathbf{v}$ and $\mathbf{w}$).

3.

(a) Determine h, so that the following vectors span $\mathbb{R}^3$.

$$\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ h \\ -6 \end{bmatrix}$$

(b) If you choose some h so that the three vectors do span $\mathbb{R}^3$, apply the Big Theorem and conclude the right statement about linear independence of these vectors.

4.

(a) Write down the precise definition of 'Linear Independence' of the vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m$.

(b) Let $\mathbf{u}$, $\mathbf{v}$ be vectors in $\mathbb{R}^n$. Then $\mathbf{u} - \mathbf{v}$ is which kind of object?

(c) Which equation do you have to consider, in order to determine the linear independence of $(\mathbf{u} - \mathbf{v})$ and $\mathbf{u} + \mathbf{v}$.

(d) Assume that $\mathbf{u}$ and $\mathbf{v}$ are linearly independent. Use this and the approach of (c) in order to show that $(\mathbf{u} - \mathbf{v})$ and $\mathbf{u} + \mathbf{v}$ are linearly independent, too.

5. Consider the following linear homomorphism:

$$T : \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \to \begin{bmatrix} u_1 + 2u_2 - u_3 \\ -4u_1 - 7u_2 + 7u_3 \\ -u_1 - u_2 + 5u_3 \end{bmatrix}.$$

(a) Find the corresponding matrix A, such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.

(b) Find the kernel of T.

(c) Find the image of

$$\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

under T.