Math 307 B – Final Exam

March 17, 2008

Name:_______________________________________

Student ID:__________________________

Directions:

- You have 110 minutes to complete this exam.
- There are 8 problems, and the exam is worth 150 points.
- You are allowed a non-graphing calculator and a sheet of notes.
- Show all your work, and put a box around your final answer.
- If you have any questions, ask me.

1	24	
2	16	
3	20	
4	10	
5	20	
6	20	
7	16	
8	24	
Total	150	

1. (a) (7 points) Find the Laplace transform of $f(t) = t^2 e^{5t} + e^{-4t}\cos(3t)$.

(b) (7 points) Find the inverse Laplace transform of $F(s) = \dfrac{3!}{(s+2)^4} + \dfrac{3}{s^2-4}$.

(c) (10 points) Find the inverse Laplace transform of $G(s) = \dfrac{2s-5}{s^2-6s+10}$.

2. (a) (6 points) Write the multipart rule for the function $f(t) = 4u_2(t) - 4u_5(t)$, and sketch its graph.

(b) (10 points) Find the Laplace transform of $g(t) = \begin{cases} t^2, & 0 \le t < 2 \\ 4, & t \ge 2. \end{cases}$

3. (20 points) Solve the following initial value problem.

$$y'' + 5y' + 6y = 4u_2(t) - 4u_5(t); \qquad y(0) = 0,\ y'(0) = 0.$$

4. (a) (5 points) Show that the Laplace transform is a linear operator.

(b) (5 points) Define an operator $\mathcal{E}xp$ by the formula $\mathcal{E}xp\left\{f(t)\right\}(s) = e^{f(s)}$. Show that $\mathcal{E}xp$ is NOT a linear operator.

5. A certain forced damped oscillator is governed by the equation

$$2u'' + 2u' + 5u = 5\cos 2t,$$

where $u(t)$ is measured in centimeters and t is measured in seconds.

(a) (5 points) What is the general form of the transient solution?

(b) (15 points) Find the steady state solution, and determine its amplitude and phase.

6. For each of the following equations, give the general form of the homogeneous (complementary) solution and the particular solution, using the method of undetermined coefficients. You do not need to solve for the constants.

   (a) (10 points) $y'' + 9y = t \cos 3t$

   (b) (10 points) $y'' - 4y' = 5t + 7t^2 e^{3t}$

7. (16 points) Find the general solution to the following equation.

$$ty' + (t+2)y = te^{-t}$$

8. A small colony of tribbles has infested your kitchen. Their population is modeled by the equation

$$\frac{dy}{dt} = -2y \ln(y/5000),$$

where y is the number of tribbles and t is measured in days.

(a) (7 points) Identify the equilibrium solutions and classify each one as stable or unstable. (Notice that the equation only makes sense for $y > 0$.) Be sure to justify your answers.

(b) (12 points) Find the general form of the solution $y(t)$. (Hint: You will want to use the substitution $u = \ln(y/5000)$.)

(c) (5 points) If the initial population is 5 tribbles, how long will it take for the tribble population to reach 4000?