Math 310: Proofs By Induction Worksheet

1. ***Prove that for all*** $n \geq 4$, $3^n \geq n^3$.

   Scratch work:

   (a) **What is the predicate P(n) that we aim to prove for all** $n \geq n_0$**?**

   $$P(n) : \underline{\hspace{6cm}}$$

   (b) **What is** $n_0 =? =$

   (c) **So the base case consists of proving** $P(n_0)$**. Write out what this means, specifically for this problem.**

   $$P(\underline{\hspace{0.5cm}}) : \underline{\hspace{6cm}}$$

   Now verify it.

   (d) **The induction step is to show that** $P(k) \Rightarrow P(k+1)$ **(for any** $k \geq n_0$**). Spell this out.**

   i. **The** ***Induction Hypothesis*** **is** $P(k)$**. Write it out.**

   $$P(k) : \underline{\hspace{6cm}}$$

   ii. **Write out the goal:** $P(k+1)$**.**

   $$P(k+1) : \underline{\hspace{6cm}}$$

   iii. **Rewrite the LHS of** $P(k+1)$ **until you can relate it to the LHS of** $P(k)$**.**

   $$3^{k+1} = \underline{\hspace{6cm}}$$

   iv. **Rewrite the RHS of** $P(k+1)$ **until you can relate it to the RHS of** $P(k)$**.**

   $$(k+1)^3 = \underline{\hspace{6cm}}$$

   v. **The induction hypothesis gives you the inequality between certain "chunks" of the RHS and LHS of** $P(k+1)$**. It remains to compare the remaining parts and show that the inequality holds between those too. Can you think of a way?**

   Use the back of the page to write a clear, correct, succint proof of the statement.

2. Prove that 7 divides $2^{n+2} + 3^{2n+1}$ for any non-negative integer n.

Scratch work:

(a) **What is the predicate P(n) that we aim to prove for all $n \geq n_0$?**

$P(n)$: ______________________

(b) **What is $n_0 =? =$**

(c) **So the base case consists of proving $P(n_0)$. Write out what this means, specifically for this problem.**

$P(\_\_)$: ______________________

Now verify it.

(d) **The induction step is to show that $P(k) => P(k+1)$ (for any $k \geq n_0$). Spell this out.**

i. **The *Induction Hypothesis* is $P(k)$. Write it out.**

$P(k)$: ____________ $= 7a$ **for some integer** a

ii. **Write out the goal: $P(k+1)$.**

$P(k+1)$: ____________ $= 7b$ **for some integer** b

iii. **Rewrite the LHS of $P(k+1)$ until you can relate it to the LHS of $P(k)$.**

iv. **Prove the induction step entirely.**

Use the back of the page to write a clear, correct, succinct proof of the statement.