This week we will discuss partially ordered sets. Please read the lecture notes as well as Chapter 2 in the book.

Written Assignments Due Friday, 1/20/17.

1. (a) Let $\leq_1$ and $\leq_2$ be two partial orders on a set X. Define a new relation $\leq$ on X by $x \leq y$ if and only if both $x \leq_1 y$ and $x \leq_2 y$ hold. Prove that $\leq$ is also a partial order.

   (b) Let $(X_1, \leq_1)$ and $(X_2, \leq_2)$ be partially ordered sets. Define a relation $\leq$ on the set $X_1 \times X_2 = \{(x_1, x_2) \ : \ x_1 \in X_1, x_2 \in X_2\}$ by $(x_1, x_2) \leq (x_1', x_2')$ if and only if $x_1 \leq_1 x_1'$ and $x_2 \leq_2 x_2'$ hold. Prove that $\leq$ is also a partial order.

2. Consider the set $[m] \times [n] = \{(i, j) \ : \ i \in [m] \text{ and } j \in [n]\}$, and let $\preceq$ be the relation on $[m] \times [n]$ defined by $(i, j) \preceq (i', j')$ if $i = i'$ and $j \leq j'$.

   (a) Prove that $\preceq$ is a partial order on $[m] \times [n]$.

   (b) Draw the Hasse diagram of the above poset, P.

   (c) Compute $\alpha(P)$. Explain your answer!

3. (a) Prove that $\omega(\mathcal{B}_n) = n + 1$. (This is #2 from §2.4.)

   (b) Let P be the poset of numbers $\{1, 2, \ldots, n\}$ ordered by divisibility. What is $\omega(P)$?

4. (a) Find the number of 2-element chains in $\mathcal{B}_n$.

   (b) Find the number of 2-element independent sets in $\mathcal{B}_n$.