This week we will discuss the Sperner theorem (see §7.2) and the Erdős-Ko-Rado theorem. Please read the lecture notes as well as take a look at §7.2 in the book.

Written Assignments Due Friday, 1/27/17.

1. Let k, l be natural numbers. Prove that every sequence of real numbers of length $kl + 1$ contains a nondecreasing subsequence of length $k + 1$ or a decreasing subsequence of length $l + 1$. (This is #4 from §2.4. **Hint:** mimic the proof of the Erdős-Szekers theorem.)

2. Let $a_1, a_2, \ldots, a_n$ be real numbers satisfying $a_i \geq 1$ (for $i = 1, 2, \ldots, n$). We want to choose as many partial sums of a_i's, i.e., expressions of the form $a_{i_1} + \cdots + a_{i_k}$, where $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ as possible so that the values of every two of chosen sums differ from each other by less than 1. Show that we cannot choose more than $\binom{n}{\lfloor n/2 \rfloor}$ such partial sums.
   Hint: if $a_{i_1} + \cdots + a_{i_k}$ and $a_{j_1} + \cdots + a_{j_p}$ are among the chosen sums, can $\{i_1, \ldots, i_k\}$ be a subset of $\{j_1, \ldots, j_p\}$?

3. Let $1 \leq k \leq n/2$ and let $\mathcal{F}$ be an independent family of subsets of $[n]$ all of whose elements have size at most k (i.e., $|F| \leq k$ for all $F \in \mathcal{F}$). Show that $|\mathcal{F}| \leq \binom{n}{k}$.
   Hint: the LYM inequality might be helpful.

4. Prove the following extensions of the LYM inequality.
   (a) Let $\mathcal{F}$ be a family of subsets of $[n]$ **not** containing any $k+1$ nested sets: $F_1 \subset F_2 \ldots \subset F_{k+1}$. Show that
   $$\sum_{F \in \mathcal{F}} \frac{1}{\binom{n}{|F|}} \leq k.$$
   Hint: note that every maximal chain in $\mathcal{B}_n$ contains at most k elements of $\mathcal{F}$.
   (b) Let $\mathcal{F}$ be a family of subsets of $[n]$ that contains **no** two elements $A \supseteq B$ with $|A - B| \geq k$. Prove that
   $$\sum_{F \in \mathcal{F}} \frac{1}{\binom{n}{|F|}} \leq k.$$