This week we will dicsuss Ramsey's theorem. Please read the lecture notes as well as §11.1 and §11.2 in the book.

**Written Assignments Due Wednesday, 2/3/17.**

1. (a) Let $\{A_1, \ldots, A_s\}$ be a collection of $l$-subsets of $[n]$, where $l < n/2$. Show that there exist $s$ distinct $(l+1)$-subsets $B_1, \ldots B_s$ of $[n]$ such that $A_i \subset B_i$ for every $1 \leq i \leq s$.
   **Hint:** Problem #5 from Homework 1 might be useful.

   (b) Prove the following extension of the Erdős-Ko-Rado theorem. Let $1 \leq k \leq n/2$ and let $\mathcal{F}$ be a family of subsets of $[n]$ all of whose elements have size at most $k$ (i.e., $|A| \leq k$ for all $A \in \mathcal{F}$). Show that if $\mathcal{F}$ is both **intersecting** and **independent** then $|\mathcal{F}| \leq \binom{n-1}{k-1}$.
   **Hint:** Use part (a) to reduce this claim to the Erdős-Ko-Rado theorem.

2. Imagine that $n$ guests attend a Christmas party. Assume that

   - in any group of three guests, there are two who do not know each other, and
   - in any group of seven guests, there are two who do know each other.

   At the end of the party, every guest gives a present to each of the guests she knows. Prove that the total number of presents given at this party is at most $6n$.

3. Show that if $k \leq k'$ and $l \leq l'$, then $r(k, l) \leq r(k', l')$.

4. Consider a group of 8 people, each pair of which are either friends or enemies. Show that if some person in the group has at least 6 friends, then there are 4 people who are mutual friends or 3 people who are mutual enemies.

5. Let $n_k = k!(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{k!}) + 1$.

   (a) Check that $(k+1)(n_k - 1) < n_{k+1} - 1$.

   (b) Consider the complete graph on $n_k$ vertices, $K_{n_k}$. Prove by induction on $k$ that if we color each edge of $K_{n_k}$ with one of the $k$ given colors, then no matter how we do this, there will always be a triangle all of whose edges receive the same color.
   **Hint:** follow the same ideas as in the proof of Theorem 11.2.1; use part (a).