Our next few lectures will be devoted to the probabilistic method. Specifically, we'll discuss random variables and expected values, as well as applications of expected values to combinatorics. Please read the lecture notes and start glancing at §10.2 (but skip the subsection "Independent events") and §§10.3, 10.4 in the book.

Written Assignments Due Friday, 2/17/17.

1. Assume you have a 6-sided dice with numbers from 1 to 6 written on its sides. Also assume that the dice is unbalanced: when you throw it, the probability to get a number k is proportional to k. What is the probability of getting either '1' or '2'?

2. A deck of cards has 52 "regular" cards and 2 jokers (54 cards total). Assume the deck is shuffled so that the cards are distributed randomly. Let p_k be the probability that, counting from the top of the deck, the first appearance of a joker happens on the kth card. Which k maximizes p_k?
   Hint: compute p_k for each k, then compare p_k and p_{k+1}.

3. Recall that a composition of n is an ordered sequence of positive integers that sum up to n. The integers appearing in a composition are called *parts* of this composition. (For instance $(3, 1, 3, 2)$ is a composition of 9; its first part is equal to 3.)

   (a) Prove that the number of all compositions of n is 2^{n-1}.
   Hint: How many compositions of n with k parts are there?

   (b) What is the probability of the event that a randomly selected composition of n has its first part equal to 1? (We assume that all compositions of n are equally likely.)