This week we'll discuss recurrence relations and generating functions. Please read the lecture notes and take a look at §§12.1-12.3 in the book.

Written Assignments Due Monday, 3/6/17.

1. Determine the number of regions that are created by n lines in the plane if it is known that (i) no two of these lines are parallel, and (ii) no three lines share a common point.

2. Use generating functions to find an explicit formula for a_n if $a_0 = 1$ and $a_{n+1} = 3a_n + 2^n$ for $n \geq 0$.

3. Let h_n equal the number of different ways in which the squares of a $1 \times n$ board can be colored, using the colors red, white, and blue, so that no two red squares are adjacent. Find a recurrence relation that h_n satisfies. Then find an explicit formula for h_n by solving this recurrence relation. **Note:** don't be surprised if your explicit formula contains square roots and doesn't look particularly pleasing.

4. Determine the generating function for the number h_n of nonnegative integral solutions of

$$2e_1 + 5e_2 + e_3 + 7e_4 = n.$$

   (Note: you do **not** need to find a formula for h_n from the generating function.)

5. Determine the generating function for the number h_n of bags of n fruit — apples, oranges, bananas, and pears — in which there are an even number of apples, at most two oranges, a multiple of three number of bananas, and at most one pear. Then find a formula for h_n from the generating function.

6. (a) Show that $\displaystyle\sum_{k=1}^{\infty} kx^k = \frac{x}{(1-x)^2}$ for $x \in (-1, 1)$.

   (b) We toss a coin n times, and record our results in a sequence ω. Set $X(\omega) = k$ if a head occurs in position k first and set $X(\omega) = 0$ if a head never occurs. Show that $E[X] < 2$. (We assume that all 2^n possible outcomes are equally likely.)
   Hint: part (a) might be of help.