In this class we will study several theorems in various areas of combinatorics that are considered by many as "gems" of the modern combinatorial theory. More specifically, we will concentrate on

1. extremal combinatorics and some of its applications;
2. linear algebra methods in combinatorics;
3. applications of Borsuk-Ulam theorem to problems in graph theory and discrete geometry.

Extremal combinatorics deals with problems of the type 'If a collection of objects satisfies certain restrictions, how large or how small can it be?'. Among the results we will study are the sunflower lemma, the Erdös-Ko-Rado theorem, the Sperner theorem, and their applications e.g. to the Littlewood-Offord problem.

Methods from Linear Algebra provide simple yet very powerful research tools in extremal combinatorics, combinatorial geometry and theory of computing. Here are several examples: Can you dissect a pyramid by a finite number of plane cuts and put the pieces together to form a cube? What is the maximum number of points in $\mathbb{R}^n$ with only two distances? Among culminations of Linear Algebra methods is a disproof of Borsuk's conjecture, which was open for about 60 years! It asserted that every subset of $\mathbb{R}^n$ of diameter 1 can be partitioned into n+1 sets of smaller diameter. We will discuss several such techniques (e.g. basic dimension arguments, spaces of polynomials and tensor product methods) and their applications.

Starting with Lovasz' striking proof of Kneser's conjecture (that asks for lower bounds on the chromatic number of a certain family of graphs), methods from algebraic topology have been employed to solve a wide variety of combinatorial problems. We will discuss a few such applications e.g. the Ham sandwich theorem, Kneser's conjecture, and topological Tverberg's theorem.

We will not follow any particular book. Instead I'll give references as we go along.

We will have occasional homework assignments. Your grades will be based on them and (possibly) on paper presentations.