Math 125 H - Winter 2015
Midterm Exam Number One
January 29, 2015

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	12	
2	10	
3	7	
4	6	
5	8	
6	5	
7	12	
Total	60	

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	–

- This exam consists of SEVEN problems on SIX pages, including this cover sheet.
- Show all work for full credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 80 minutes to complete the exam.

–

1. **[4 points per part]** Evaluate each integral. You may use any techniques you know.

(a) $\int \left(e^{3x} - \sin^2(x)\cos(x)\right) dx$

(b) $\int \frac{\sec^2(\ln(x^2))}{x}\, dx$

(c) $\int x^3\sqrt{x^2+4}\, dx$

2. **[10 points]** A particle is moving along the x-axis.

   At time $t \geq 0$ seconds, its acceleration is given by $a(t) = 2t - 8$.

   At time $t = 0$ it's at $x = 3$, and at time $t = 3$ it's at $x = 21$.

   In the first 8 seconds, what is the **total distance traveled** by the particle?

3. **[7 points]** Let $\mathcal{R}$ be the region bounded by $y = \ln(x+1)$, the y-axis, and the line $y = 2$. Compute the volume of the solid obtained by revolving $\mathcal{R}$ around the y-axis.

4. **[6 points]** Use any techniques you'd like to compute the following limit:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \tan\left(\frac{i\pi}{3n}\right) \frac{\pi}{12n}$$

5. **[8 points]** Compute the area of the region bounded by $y = x$, $y = \dfrac{x+1}{x^2+1}$, and the y-axis.

6. **[5 points]** Consider the following function $f(x)$:

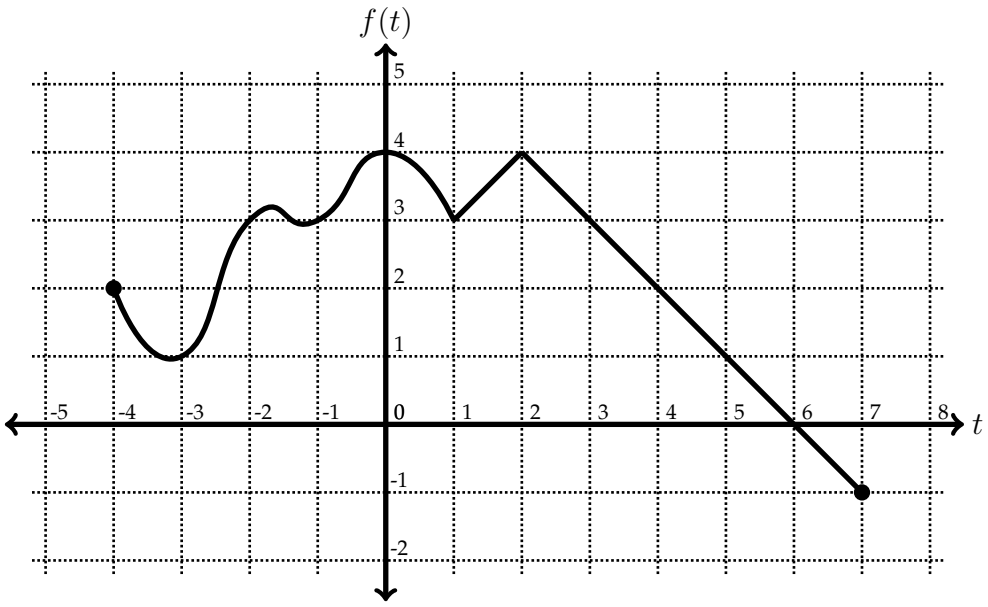
$$f(x) = \begin{cases} 2 + \sqrt{9 - x^2} & \text{if } 0 \leq x \leq 3 \\ 8 - 2x & \text{if } 3 < x \leq 4 \end{cases}$$

Compute $\displaystyle\int_0^4 f(x)\,dx$.

7. **[4 points per part]** You awake to a large commotion outside your window.

   "It's the graph question," cries a youth. "Jonah wrote another graph question!"

   A church bell chimes, and a parade makes its way through the plaza. This is a good day.



(a) Let $h(x) = \int_{e^x}^{x} \cos(\pi t) f(t)\, dt$. Compute $h'(0)$.

(b) Compute $\int_1^2 2^t f(2^t + 1)\, dt$.

(c) Suppose the area under $f(t)$ from -4 to 2 is revolved around the horizontal axis.

Write (but don't evaluate) an integral for the resulting volume, and compute the M_3 approximation of that integral.