Project: Markov Chains

General Information. *Markov chains* are designed to model systems that change from state to state. In particular, the current state should depend only on the previous state. For example, a city's weather could be in one of three possible states: sunny, cloudy, or raining (note: this can't be Seattle, where the weather is *never* sunny. Here, linear algebra is used to predict future conditions.

Key Words. Stochastic process, Markov chains, transition probabilities, state vectors, steady-state vectors.

References. Markov chains are often mentioned in books about probability or stochastic processes. These books may be a bit beyond what you've previously been exposed to, so ask for help if you need it.

Problems.

(1) For the transition matrix $P = \begin{bmatrix} .4 & .5 \\ .6 & .5 \end{bmatrix}$,

(a) calculate the first 5 state vectors if the initial state vector is $(1 \ \ 0)^T$.

(b) Find the steady-state vector of P.

(2) Jim is either happy or pouting (a simple soul, our Jim). If he is happy one day, he is happy the next day four times out of five. If he is pouting one day, the chances that he will also pout the next day are one time out of three. Over the long term, what are the chances that Jim is happy on any given day?

(3) A mouse trap is placed in room 1 of the house with the pictured floor plan. Each time the mouse comes into room 1, he is trapped with probability $p = .1$. If he is not trapped, he leaves each room by one of its exits, chosen at random. Model the path of the mouse through the house as a Markov chain.