October 21, 2016

Announcements

- Assigned reading for the week sections 6.3, Midterm review, 6.4.
- Homework #4A Due Wednesday, October 26, 11:00pm. You should aim to have completed this portion by Monday evening, October 24.
- Quiz #3 (taken from HW #4A) Tuesday, October 25 in TA sections
- Homework #4B and 4C Due Friday, October 28, 11:00pm
- Midterm #1 was Yesterday
  - Midterms will be returned in Quiz section next Tuesday.
  - Solutions will also be distributed.

Today

- 6.4: Work

Work: If an object moves along a straight line with position $s(t)$ then the force F on the object (in the same direction) is defined by Newton's Second Law of Motion

$$F = m\,\frac{d^2s}{dt^2}$$

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$(Newton) = (kg) \times (m/s^2)$$

When the acceleration is constant so is the force F. We then define the work to be the product of the force F and the distance d that the object moves:

$$W = Fd$$

$$\text{work} = \text{force} \times \text{distance}$$

$$(Joule) = (Newton) \times (meter)$$

• Careful: Weight is a measurement of force (so there is no need to multiply it by g, the acceleration due to gravity).

Suppose that an object moves along the x- axis in the positive direction, from $x = a$ to $x = b$, and that at each point x between a and b a force $f(x)$ acts on the object, where $f(x)$ is a continuous function. To estimate the work done we divide $[a, b]$ into n-subintervals with end points x_0, x_1,...,x_n and equal width Δx. Choose sample points $x_i^* \in [x_{i-1}, x_i]$. For large n, Δx is small and the values of f in $[x_{i-1}, x_i]$ are very close to $f(x_i^*)$. The work W_i that is done in moving the particle from x_{i-1} to x_i is approximately

$$W_i \sim f(x_i^*)\Delta x.$$

Thus the total work done in moving the particle from a to b is approximately

$$W \sim \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

Letting $n \to \infty$ we have

$$W = \int_a^b f(x)\, dx.$$

Hooke's law

The force required to maintain a spring stretched x units beyond its natural length is proportional to x:

$$f(x) = kx$$

$k > 0$ is the spring constant.

Example 1: (6.4 #8) A spring has natural length 20cm. If a 25N force is required to keep it stretched to a length of 30cm, how much work is required to stretch it from 20cm to 25cm?

Example 2: A circular reservoir has diameter 24m, the sides are 5m high, and the depth of the water is 4m. How much work is required to pump all the water out over the side? (Use the fact that the density of the water is $1000\,\mathrm{kg/m^3}$.)

Example 3: A leaky 10kg bucket is lifted from the ground to a height of 12m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12m level. How much work is done?