Announcements

- Assigned reading for the week: sections 6.5, 7.1 and 7.2
- Homework #4A Due Wednesday, October 26, 11:00pm. You should aim to have completed this portion by tonight.
- Quiz #3 (taken from HW #4A) Tuesday, October 25 in TA sections
- Homework #4B and 4C Due Friday, October 28, 11:00pm
- Midterm #1: median $= 40$ and mean $= 38.3$ and the standard deviation was 8.9.
  - 234 exams graded. 9 perfect scores 50/50!
  - Midterms will be returned in Quiz sections Tomorrow.
  - Solutions will also be distributed.
  - Requests for corrections to arithmetic errors in grading must be made in writing and handed in, with the exam, to either me or your TA this week. Please do not make requests for "more partial credit".

Today

- 6.4: Work (Example 3 from Friday)
- 6.5: Average Value of a Function

Example 3: A leaky 10kg bucket is lifted from the ground to a height of 12m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12m level. How much work is done?

Solution to Example 3 (from Friday): A leaky 10kg bucket is lifted from the ground to a height of 12m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12m level. How much work is done?

Let x denote the height above the ground. So that $0 \leq x \leq 12$.

At height x we compute the 3 contributions to the mass:

- The mass of the rope is given by the density of the times the length of rope that is being pulled at height x, this is $= 0.8(12 - x)$
- The fact that the bucket is leaking at constant speed, means the amount of water $m(x)$ present is a linear function of the height x:

$$\text{slope} = \frac{m(x) - m(12)}{x - 12} = \frac{36 - 0}{0 - 12} = -3$$

  So, the mass of the water is given by $m(x) = 3(12 - x)$
- The mass of the bucket is constant $= 10$

The total mass to be lifted at height x is given by the sum:

$$0.8(12-x)+3(12-x)+10=55.6-3.8x \text{ kg}.$$

The total force which must be applied is equal and opposite to the gravitational force (recall $F=ma$) on this mass. Thus the force applied to lift this is: $9.8(55.6-3.8x)$ N.

The infinitessimal work to move this mass, at height x, a distance dx, is thus

$$9.8(55.6-3.8x)\,dx$$

So that the total work is given by the integral

$$\begin{aligned} W &= 9.8\int_0^{12}(55.6-3.8x)\,dx \\ &= 9.8(55.6x-1.9x^2)\,|_0^{12} \\ &= 9.8(667.2-273.6) \\ &= 3857.28 \text{ Joules }. \end{aligned}$$

The Average Value of a Function

Definition: Let f be a continuous function on $[a, b]$. The average value of f on $[a, b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Mean Value Theorem for Integrals

Mean Value Theorem for Integrals: Let f be a continuous function on $[a, b]$. There exists $c \in [a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a).$$

or

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

where

$$f(c) = \text{ the average value of the function } f(x)$$

on the interval $[a, b]$.

Example 2: If a coffee cup has temperature 95°C in a room where the temperature is 20°C, then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is

$$T(t) = 20 + 75e^{-t/50}.$$

What is the average temperature of the coffee during the first half an hour?