October 28, 2016

Announcements

- Assigned reading for the week: sections 6.5, 7.1 and 7.2
- Homework #4B and 4C Due tonight Friday, October 26, 11:00pm
- Homework #5 (125 HW # 5ABC, all 3 parts) Due Wednesday, November 2, 11:00pm (complete before section, Tuesday 11/1)
- Quiz #4 (taken from HW #4BC and/or #5AB) Tuesday, November 1 in TA sections
- Midterm #1:
  - Requests for corrections to arithmetic errors in the midterm grade must be made in writing and handed in, with your exam, to either me or your TA by the end of today.

Today

- 7.2 Trigonometric Integrals

Trigonometric identities

- $\sin^2 x + \cos^2 x = 1$
- $\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \dfrac{1}{2}(1 + \cos 2x)$
- $\sin x \cos x = \dfrac{1}{2}\sin 2x$
- $\sin a \cos b = \dfrac{1}{2}[\sin(a-b) + \sin(a+b)]$
- $\sin a \sin b = \dfrac{1}{2}[\cos(a-b) - \cos(a+b)]$
- $\cos a \cos b = \dfrac{1}{2}[\cos(a-b) + \cos(a+b)]$

Trigonometric Integrals

Strategy for evaluating $\displaystyle\int \sin^m x \, \cos^n x \; dx$

(1) If $n = 2k + 1$ then

$$\begin{aligned} \int \sin^m x \, \cos^{2k+1} x \, dx &= \int \sin^m x \, (\cos^2 x)^k \, \cos x \, dx \\ &= \int \sin^m x \, (1 - \sin^2 x)^k \, \cos x \, dx \end{aligned}$$

Then substitute $u = \sin x$ (so that $du = \cos x \, dx$) and integrate

$$\int u^m \, (1 - u^2)^k du.$$

(2) If $m = 2k + 1$ then

$$\begin{aligned}\int \sin^{2k+1} x \cos^n x \, dx &= \int (\sin^2 x)^k \cos^n x \sin x \, dx \\ &= \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx\end{aligned}$$

Then substitute $u = \cos x$ (so that $du = -\sin x \, dx$) and integrate

$$-\int u^n (1 - u^2)^k du.$$

(3) If $m = 2k$ and $n = 2j$ then

$$\int \sin^{2k} x \cos^{2j} x \, dx = \int \left(\frac{1 - \cos 2x}{2}\right)^k \left(\frac{1 + \cos 2x}{2}\right)^j dx.$$

Secant Integrals

We showed that

$$\int \sec x \, dx = \frac{1}{2} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right| + C$$

Note that the book uses a different derivation than we gave and they arrive at the formula

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

These two expressions appear very different however they are both correct.

We also may need

$$\int \sec^3 x \, dx \quad = \quad \frac{1}{2} \sec x \tan x + \frac{1}{4} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right| + C$$