November 2, 2016

Announcements

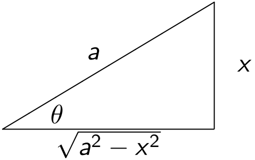
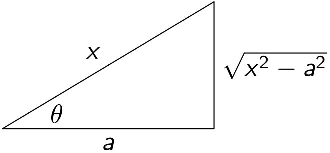
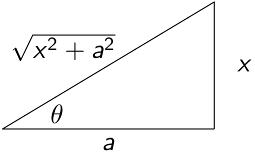
- Assigned reading for the week: sections 7.3, 7.4 and 7.5
- Printout and bring the Worksheet "Algebra and Partial Fractions" with you tomorrow to your TA sections
- Homework #5 (125 HW # 5ABC, all 3 parts) due Tonight 11:00pm
- Homework # 6AB Due Wednesday, November 9, 11:00pm (complete before quiz section, Tuesday 11/8)
- Quiz #5 (from HW # 5C & HW #6AB) Tuesday, November 8 in TA sections
- Extra Practice Problems for integration are available via a link from the Week 6 Outline on the Math 125 Materials page (view this as homework that you should do but do not have to turn in):

  http://www.math.washington.edu/~m125/Homeworks/week6ExtraPracticeProbs.pdf

Today

- Finish 7.3 Trigonometric Substitution
- 7.4 Partial Fractions

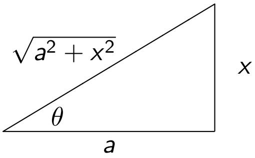
Trigonometric Substitution

Expression (Not always "$\sqrt{}$")	**Triangle**	**Substitution**
$\sqrt{a^2 - x^2}$		$x = a \sin\theta$
$\sqrt{x^2 - a^2}$		$x = a \sec\theta$
$\sqrt{x^2 + a^2}$		$x = a \tan\theta$

Expression

$$\sqrt{a^2 + x^2}$$

Triangle



Substitution

$$x = a \tan\theta \quad \text{with} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \qquad \Longrightarrow \qquad dx = a \sec^2\theta \, d\theta.$$

Identity

$$1 + \tan^2\theta = \sec^2\theta = \frac{1}{\cos^2\theta}$$

Useful relationships

$$\theta = \arctan\left(\frac{x}{a}\right) \qquad \cos\theta = \frac{a}{\sqrt{a^2 + x^2}} \qquad \sin\theta = \frac{x}{\sqrt{a^2 + x^2}}$$

Recall Secant Integrals:

$$\int \sec x \, dx = \frac{1}{2} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right| + C$$

or

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

and

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{4} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right| + C$$

Exercise 1: Compute

$$\int \frac{dt}{(t^2 - 6t + 13)^{1/2}}$$

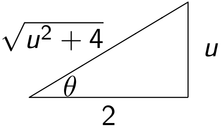
First Complete the Square:

$$t^2 - 6t + 13 = t^2 - 6t + 9 + 4 = (t-3)^2 + 4.$$

Now use the substitution $u = t - 3$ so that $du = dt$, this gives

$$\int \frac{dt}{((t-3)^2 + 4)^{1/2}} = \int \frac{du}{(u^2+4)^{1/2}}.$$

Triangle:



So we do the Trig substitution $u = 2\tan\theta$, and therefore
$du = 2\sec^2\theta\, d\theta = 2(1+\tan^2\theta)\, d\theta$.

The integral then becomes

$$
\begin{aligned}
\int \frac{2(1+\tan^2\theta)\,d\theta}{(4\tan^2\theta+4)^{1/2}} &= \int \sqrt{1+\tan^2\theta}\,d\theta \\
&= \int \sec\theta\,d\theta \\
&= \frac{1}{2}\ln\left|\frac{1+\sin\theta}{1-\sin\theta}\right| + C \\
&= \frac{1}{2}\ln\left|\frac{1+\frac{u}{\sqrt{u^2+4}}}{1-\frac{u}{\sqrt{u^2+4}}}\right| + C \\
&= \frac{1}{2}\ln\left|\frac{\sqrt{u^2+4}+u}{\sqrt{u^2+4}-u}\right| + C, \qquad \text{so} \\
\int \frac{dt}{(t^2-6t+13)^{1/2}} &= \boxed{\frac{1}{2}\ln\left|\frac{\sqrt{t^2-6t+13}+(t-3)}{\sqrt{t^2-6t+13}-(t-3)}\right| + C}
\end{aligned}
$$