November 4, 2016

Announcements

- Assigned reading for the week: sections 7.3, 7.4 and 7.5
- Next Friday, November 11 is Veterans Day - No Classes
- Homework # 6AB Due Wednesday, November 9, 11:00pm (complete parts A & B before quiz section, Tuesday 11/16)
- Quiz #5 (taken from HW # 5C and HW #6AB) Tuesday,November 8 in TA sections
- Extra Practice Problems for integration are available via a link from the Week 6 Outline on the Math 125 Materials page (view this as homework that you should do but do not have to turn in):

  http://www.math.washington.edu/~m125/Homeworks/week6ExtraPracticeProbs.pdf

Today

- 7.4 Partial Fractions
- 7.5 Strategy for Integration

A rational function is a function such that

$$f(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials, i.e.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

$$Q(x) = b_m x^m + b_{m-1} x^{m-1} + \cdots + b_0$$

with $a_n \neq 0$ and $b_m \neq 0$. The degree of P is n and the degree of Q is m.
Notation: $\deg(P) = n$ and $\deg Q = m$.
If $\deg(P) \geq \deg Q$ divide P by Q to obtain a remainder R such that $\deg(R) < \deg Q$, i.e.

$$f(x) = \frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)},$$

where R and S are also polynomials.

Steps to integrate a rational function of the form

$$\frac{R(x)}{Q(x)} \text{ with } \deg(R) < \deg Q.$$

Step 1: Factor Q as a product of factors of the form

$$ax + b$$

or

$$ax^2 + bx + c \text{ with } b^2 - 4ac < 0.$$

Step 2: Express $\frac{R(x)}{Q(x)}$ as a sum of **partial fractions** of the form

$$\frac{A}{(ax+b)^j} \text{ or } \frac{Ax+B}{(ax^2+bx+c)},$$

with $b^2 - 4ac < 0$.

Case I: Q is a product of distinct linear factors, i.e.

$$Q(x) = (a_1x + b_1)(a_2x + b_2)\cdots(a_lx + b_l),$$

then

$$\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \cdots + \frac{A_l}{a_lx + b_l} \tag{1}$$

Case II: Q is a product of linear factors some of which are repeated. Suppose $(a_1x + b_1)$ is repeated j_1 times. Then the term

$$\frac{A_1}{a_1x + b_1}$$

in (1) is replaced by

$$\frac{A_{1,1}}{(a_1x + b_1)} + \cdots + \frac{A_{1,j_1}}{(a_1x + b_1)^{j_1}} \tag{2}$$

Case III: Q contains irreducible quadratic factors, none of which is repeated. Then in addition to the terms that appear in (1) and (2) the expression for $\frac{R(x)}{Q(x)}$ will contain a term of the form

$$\frac{Ax+B}{ax^2+bx+c} \tag{3}$$