November 7, 2016

Announcements

- Assigned reading for the week: sections 7.5, 7.7 and 7.8
- Homework # 6AB Due Wednesday, November 9, 11:00pm (complete parts A & B before quiz section, Tuesday 11/16)
- Quiz #5 (taken from HW # 5C and HW #6AB) Tuesday,November 8 in TA sections
- Extra Practice Problems for integration are available via a link from the Week 6 Outline on the Math 125 Materials page (view this as homework that you should do but do not have to turn in):

  http://www.math.washington.edu/~m125/Homeworks/week6ExtraPracticeProbs.pdf
- Friday, November 11 is Veterans Day - No Classes

Today

- 7.5 Strategy for Integration

Table of indefinite integrals

- $\displaystyle \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ and $\displaystyle \int \frac{1}{x}\,dx = \ln|x| + C$
- $\displaystyle \int e^x\,dx = e^x + C$ and $\displaystyle \boxed{\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C}$
- $\displaystyle \int \sin x\,dx = -\cos x + C$ and $\displaystyle \int \cos x\,dx = \sin x + C$
- $\displaystyle \int \sec^2 x\,dx = \tan x + C$ and $\displaystyle \int \frac{1}{\sin^2 x}\,dx = -\frac{\cos x}{\sin x} + C$
- $\displaystyle \int \sec x\,dx = \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$ and $\displaystyle \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$

Note: On exams you may use (without justification) any of the 20 integrals in the table on p. 495 of the text. You must show all your work in evaluating any other integrals (with the exception of $\sec x$ and $\sec^3 x$) even if they are on your note sheet.

Steps to evaluate an integral

1. Simplify the integrand
2. Does substitution work?
3. Does integration by parts work?
4. Classify the integrand:
   - Trigonometric function
   - Rational function
   - Contains radicals
5. Take a second look: either substitution or integration by parts must work (maybe after manipulation of the integrand).

Integration strategy

You might want to follow these steps when evaluating an integral. Be prepare to abandon an attempt and proceed with another. Do not get discouraged. It takes lots of practice to master all the techniques.

1. Simplify the integrand. Keep in mind that it is easier to integrate a sum than a product.
2. Attempt an easy substitution. General form:

$$\int f(g(x))g'(x)\,dx \quad \text{substitution :} \quad u = g(x).$$

Examples:

- $\displaystyle\int x^2\sqrt{x^3+1}\,dx$ (substitution $u = x^3 + 1$).
- $\displaystyle\int (\sin x)e^{\cos x}\,dx$ (substitution $u = \cos x$).
- $\displaystyle\int \frac{\ln x}{x}\,dx$ (substitution $u = \ln x$).

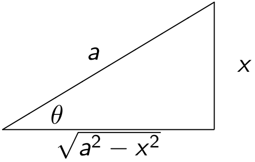
Trigonometric Substitution

Expression
(Not always "$\sqrt{}$")

Triangle

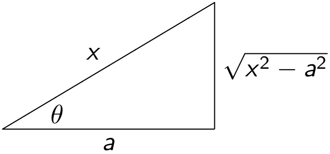
Substitution

$\sqrt{a^2 - x^2}$



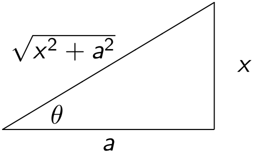
$x = a \sin \theta$

$\sqrt{x^2 - a^2}$



$x = a \sec \theta$

$\sqrt{x^2 + a^2}$



$x = a \tan \theta$

Attempt a trigonometric substitution

- General forms:
  - Expression $\sqrt{a^2 - x^2}$, substitution $x = a\sin\theta$.
  - Expression $a^2 + x^2$, substitution $x = a\tan\theta$.
  - Expression $\sqrt{x^2 - a^2}$, substitution $x = a\sec\theta$.

Examples:

- $\displaystyle\int \frac{1}{\sqrt{49 - 9x^2}}\,dx$, substitution $x = \frac{7}{3}\sin\theta$.
- $\displaystyle\int \frac{1}{(x^2 + 25)^4}\,dx$, substitution $x = 5\tan\theta$.
- $\displaystyle\int (x^2 - 16)^{5/2}\,dx$, substitution $x = 4\sec\theta$.

Attempt a rationalizing substitution

Although there is no general form in this case, the idea is to substitute the terms that bothers you the most in terms of being able to evaluate the integral.

Examples:

- $\displaystyle\int e^{x^{1/3}}\,dx,$

  substitution $u = x^{1/3}$, i.e. $u^3 = x$.

- $\displaystyle\int \frac{e^{2x}}{e^{2x} + 3e^x + 2}\,dx,$

  substitution $u = e^x$.

- $\displaystyle\int \frac{\sin x}{(\cos^2 x + 4)(\cos x + 3)}\,dx,$

  substitution $u = \cos x$.

Rational functions

We use long division (if necessary) then the method of partial fractions.

Recall:

- If $f(x) = \frac{P(x)}{Q(x)}$ where $\deg P \geq \deg Q$, than you must first carry out long division of polynomials.

- If $f(x) = \frac{R(x)}{Q(x)}$ where $\deg R < \deg Q$, the method of Partial Fractions works in the cases where Q is a product of linear factors (including repeated factors) as well as when there is a single irreducible quadratic factor in the denominator.

Integration by parts

General form

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

or

$$\int u\,dv = uv - \int v\,du.$$

Examples:

- $\int x^n \cos x\,dx$.
- $\int x^n \ln(x+3)\,dx$
- $\int x^n e^{4x}\,dx$.

Do Not Despair!

- If everything above fails, take a second look. Remember that either substitution or integration by parts must work maybe after manipulation of the integrand (for example: check to see if a trigonometric identity can be applied)