Announcements

- Assigned reading for the week: sections 7.5, 7.7 and 7.8
- Homework # 7A & 7B Due Wednesday, November 16, 11:00pm
- No quiz next week.
- **Midterm # 2, Thursday, November 17** (Covers through section 7.8)
  - Start doing practice Midterms (beware: some cover material up to 8.3).
  - One 8.5 x 11 handwritted sheet of notes (both sides)
  - You may use any of the 20 integrals from the table on p. 495 without justification. Must show your work in evaluating any other integrals.
- Friday, November 11 is Veterans Day - No Classes.
- See this page for information on the Final Exam (e.g. Official UW conflicts): *www.math.washington.edu/Undergrad/groundrules/groundrules124-5aut16.php*

Today

- 7.7 Approximate Integration: Simpson's Rule
- 7.8 Improper Integrals: How do you compute the area of unbounded regions?

Approximate Integration: Simpson's Rule

What can you do when you are asked to estimate

$$\int_0^1 e^{x^2}\,dx \qquad \text{or} \qquad \int_0^1 \cos(x^3)\,dx \ ?$$

Improper Integrals

How do you compute the area of unbounded regions?

$$\int_1^\infty \frac{1}{x^4}\,dx \qquad \text{or} \qquad \int_0^1 \frac{1}{\sqrt{x}}\,dx \ ?$$

Let f be a continuous function on $[a, b]$. Divide $[a, b]$ into n subintervals of width Δx. Hence

$$\Delta x = \frac{b-a}{n}$$

and

$$x_0 = a, \qquad x_n = b,$$

$$x_i = x_{i-1} + \Delta x = a + i\Delta x \ \text{ for } \ 1 \le i \le n.$$

Recall

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

where x_i^* is any sample point in the i^{th}-interval $[x_{i-1}, x_i]$.

Midpoint and Trapezoidal Rule

Midpoint Rule

$$\int_a^b f(x)\,dx \sim \mathbf{M_n} = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)\right]$$

where

$$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$$

Trapezoidal Rule

Instead of using Rectangles use Trapezoids whose top side is given by the cord between the endpoints x_{i-1} and x_i of the i^{th}-interval.

$$\begin{aligned}
\int_a^b f(x)\,dx &\sim \mathbf{T_n} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathrm{Area}(\mathrm{T_i}) \\
&= \sum_{i=1}^{n} \frac{[f(x_{i-1}) + f(x_i)]}{2}\Delta x \\
&= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]
\end{aligned}$$

Example 1: Apply the Trapezoidal rule to estimate

$$\int_{-1}^{1} \sqrt{1-x^2}\, dx \qquad \text{with} \qquad n = 4$$

and compare this with the precise answer.

Simpson's rule (note: this requires n to be even!)

Replace the linear approximation by a quadratic approximation.
Use each subsequent PAIR of intervals to determine the best fitting quadratic equation ($y = Ax^2 + Bx + C$) between the three points determined by the endpoints and midpoint of the pair. Add up the areas

$$A = \frac{\Delta x}{3}[f(x_{i-1}) + 4f(x_i) + f(x_{i+1})]$$

under the graph of this quadratic equation (parabola).

$$\int_a^b f(x)\,dx \sim S_n$$

$$\begin{aligned} S_n &= \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots \\ &\qquad +2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] \end{aligned}$$

Example 2: Use Simpson's rule to estimate

$$\int_{-1}^{1} \sqrt{1 - x^2}\, dx \qquad \text{with} \qquad n = 4$$

and compare this with the precise answer.