September 30, 2016

Announcements

- Keep on familiarizing yourself with the class website, the Math 125 Materials Web site and the links you find on those pages.
- The only Calculator you are allowed to use is the Ti-30x IIS.
- Assigned reading for the week sections 4.9, 5.1 and 5.2.
- Homework #1 (125 HW 1ABC, all 3 parts) should be completed by Monday night, October 3, Due Wednesday, October 5, 11:00pm.
- Quiz #1 (taken from HW #1ABC) on Tuesday, October 4 in TA sections.

Today

- 4.9: More on Antiderivatives
- 5.1: Areas and Distances
- 5.2: The definite integral

Definition

A function F is called an **antiderivative** *of f on an interval I if $F'(x) = f(x)$ for all x in I.*

Theorem

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Function	Particular antiderivative
$x^n \ (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln \lvert x \rvert$
e^x	e^x
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\sec^2 x$	$\tan x$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x = \sin^{-1}(x)$
$\frac{1}{1+x^2}$	$\arctan x = \tan^{-1}(x)$

Example: A car braked with a constant deceleration of 16 $\mathrm{ft/s^2}$, producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

Example: A car braked with a constant deceleration of 16 ft/s^2, producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

Deceleration means negative acceleration, so $a(t) = -16$.

Let's assume $t = 0$ is the time the car first applies the brakes. Then, since **velocity is the antiderivative of acceleration**,

$$v(t) = -16t + v_0$$

where v_0 the the speed at which the car is going when the brakes were applied. (Note that $v_0 = v(0)$ is the quantity we are looking for!)

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Let $s(t)$ be the position of the car measured from when the brakes are first applied (so that $s(0) = 0$)

Since **position is the antiderivative of velocity**, we have

$$s(t) = -8t^2 + v_0 t + s_0$$

where $s_0 = s(0) = 0$.

So

$$s(t) = -8t^2 + v_0 t.$$

Let t_s be the time the car stops ($v(t_s) = 0$) then

$$s(t_s) = -8t_s^2 + v_0 t_s = 200.$$

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But $v(t_s) = -16t_s + v_0 = 0$ so $v_0 = 16t_s$. Therefore

$$-8t_s^2 + 16t_s^2 = 8t_s^2 = 200.$$

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From this we may conclude that $t_s = 5 \text{ sec}$, and thus

$$\boxed{v_0 = 16t_s = 16 \times 5 = 80 \text{ ft/sec.}}$$

Area Problem:

Find the area of the region S bounded by the graph of a continuous function f (where $f(x) \geq 0$), the x-axis and the vertical lines $x = a$ and $x = b$.

$$S = \{(x, y) : a \leq x \leq b,\ 0 \leq y \leq f(x)\}$$

Idea: Divide S into n strips of the same width,

$$\Delta x = \frac{b-a}{n}$$

There is a corresponding division of $[a, b]$ into n subintervals:

$$[x_0, x_1],\ [x_1, x_2], \cdots,\ [x_{n-1}, x_n]$$

where

$$x_0 = a, \qquad x_n = b,$$

$$x_{k+1} = x_k + \Delta x \ \text{ for } \ 0 \leq k \leq n-1$$

Compute R_n and L_n:

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$$

$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x$$

Definition

The area of the region S bounded by the graph of a non-negative continuous function f, the x-axis and the vertical lines $x = a$ and $x = b$ is

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n.$$

The velocity graph of a car accelerating from rest to a speed of 120 km/h over a period of 30 seconds is shown. Estimate the distance traveled during this period.