Announcements

- This week: 7.8, Midterm Review and 8.1
- Homework # 7AB Due Wednesday, November 16 11:00pm
- No quiz this week - tomorrow Midterm review
- **Midterm # 2, Thursday, November 17** (Covers through section 7.8)
  - Start doing practice Midterms (beware: some cover material up to 8.3).
  - One 8.5 x 11 handwritted sheet of notes (both sides)
  - The only calculator which may be used is the Ti-30x IIS.
  - You may use any of the 20 integrals from the table on p. 495 without justification. Must show your work in evaluating any other integrals even if they are on your note sheet.
  - CLUE Midterm review: Tuesday, Nov 15, 6:30-8:00 pm MGH 389

Today

- 7.8 Improper Integrals: How do you compute the area of unbounded regions?

Definition of Improper Integral, Type I

1. If $\int_a^t f(x)\,dx$ exists for every $t \geq a$, then

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx,$$

provided this limit exists (as a finite number)

2. If $\int_t^b f(x)\,dx$ exists for every $t \leq b$, then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t\to-\infty} \int_t^b f(x)\,dx,$$

provided this limit exists (as a finite number).

The improper integrals

$$\int_a^{\infty} f(x)\,dx \qquad \text{and} \qquad \int_{-\infty}^{b} f(x)\,dx$$

are called **convergent** if the corresponding limits exists and **divergent** if the limit does not exists.

1. If both $\int_a^{\infty} f(x)\,dx$ and $\int_{-\infty}^{a} f(x)\,dx$ are convergent we define

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$$

Proposition:

- $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converges if $p > 1$ and we have $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx = \frac{1}{p-1}$

and

- $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ diverges if $p \leq 1$.

Comparison Theorem: Suppose that f and g are continuous functions with
$0 \leq g(x) \leq f(x)$ for $x \geq a$.

1. If $\displaystyle\int_a^\infty f(x)\,dx$ converges so does $\displaystyle\int_a^\infty g(x)\,dx$.
2. If $\displaystyle\int_a^\infty g(x)\,dx$ diverges so does $\displaystyle\int_a^\infty f(x)\,dx$.

Definition of Improper Integral, Type II

1. If f is continuous on $[a, b)$ and it is discontinuous at b, then

$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$$

   if this limit exits (as a finite number).

2. If f is continuous on $(a, b]$ and it is discontinuous at a, then

$$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$$

   if this limit exits (as a finite number).

The improper integral $\int_a^b f(x)\,dx$ is called **convergent** if the corresponding limit exists and **divergent** if the limit does not exist.

1. If f has a discontinuity at c where $a < c < b$, and both $\int_a^c f(x)\,dx$ and $\int_c^b f(x)\,dx$ are convergent, then we define

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$