November 18, 2016

Announcements

- This week: 7.8, Midterm Review and 8.1
- Homework # 8A & 8B Due Wednesday, November 23, 11:00pm
- No Quiz next Tuesday!
- Midterm # 2 will be returned in Quiz sections on Tuesday, November 22
- Next week we will focus on 8.3 Center of Mass

Today

- 8.1 Arc Length: What does length of a curve mean? How do you compute it?

The arc length formula

If f' is continuous on $[a, b]$, then the length of the curve $\{(x, y) : a \leq x \leq b,\ y = f(x)\}$, is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx,$$

or

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

If g' is continuous on $[c, d]$, then the length of the curve $\{(x, y) : c \leq y \leq d,\ x = g(y)\}$, is

$$L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy,$$

or

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy.$$

The arc length function s of the curve

$$C = \{(x, y) : a \leq x \leq b,\ \ y = f(x)\}$$

is the distance along the curve C from the initial point $P_0 = (a, f(a))$ to $Q = (x, f(x))$, i.e.

$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt.$$

By the FTC

$$\frac{ds}{dx} = \sqrt{1 + [f'(x)]^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

Thus

(1) $$(ds)^2 = (dx)^2 + (dy)^2\,.$$

Symbolically

$$L = \int ds.$$

If the curve C is given parametrically by $C = \{(x(t), y(t)) : t_0 \leq t \leq t_1\}$, then (1) becomes

$$\left(\frac{ds}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2.$$

and

$$\frac{ds}{dt} = \sqrt{(x'(t))^2 + (y'(t))^2}.$$

In this case

$$L = \int_{t_0}^{t_1} \frac{ds}{dt}\, dt = \int_{t_0}^{t_1} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt.$$

Problem A (Seattle) Seahawk flying at 15 meters per second at an altitude of 180 meters accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}$$

until it hits the ground. Here y is the height above the ground and x the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground.