Announcements

- This week 8.3 (Center of Mass)
- **Note:** Hydrostatic pressure will NOT be covered (or tested on)
- Homework # 8A & 8B Due Wednesday, November 23, 11:00pm
- Homework # 9A (Center of Mass) & 9B (Separable differential equations) Due Wednesday, November 30, 11:00pm
- No Quiz Tomorrow (Happy Thanksgiving!)
- Midterm #2: 232 exams, median = 35 and mean = 35.05
  - Midterms will be returned in Quiz sections Tomorrow.
  - Solutions will also be distributed.
  - There were 7 perfect scores: 50/50 (and numerous 49s and 48s).
  - Requests for corrections to arithmetic errors in the midterm grade must be made in writing and handed in, with your exam, to either me or your TA on Tuesday or Wednesday

Today

- Seahawk Arclength problem
- 8.3 Center of Mass

Problem A Seahawk flying at 15 meters per second at an altitude of 180 meters accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}$$

until it hits the ground. Here y is the height above the ground and x the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground.

Center of Mass: Finitely many points on a line

Consider a system of n particles with masses $m_1, m_2, \cdots, m_n$ located at points $x_1, x_2, \cdots, x_n$ on the x-axis. The center of mass of the system is located at

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i x_i}{m},$$

where $m = \sum_{i=1}^{n} m_i$ is the total mass of the system. The sum of the individual moments

$$M = \sum_{i=1}^{n} m_i x_i$$

is called the **moment of the system about the origin**. Note that

$$m\bar{x} = M.$$

Center of Mass: Finitely many points in the plane

Consider a system of n particles with masses $m_1, \; m_2, \cdots, \; m_n$ located at points (x_1, y_1), (x_2, y_2), $\cdots$, (x_n, y_n). The **moment of the system about the y-axis** is

$$M_y = \sum_{i=1}^{n} m_i x_i,$$

and the **moment of the system about the x-axis** is

$$M_x = \sum_{i=1}^{n} m_i y_i.$$

The coordinates of the center of mass $(\bar{x}, \bar{y})$ are

$$\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m},$$

where $m = \sum_{i=1}^{n} m_i$ is the total mass of the system. Note also that

$$m\bar{x} = M_y \quad \text{and} \quad m\bar{y} = M_x.$$

Example

The masses m_i are located at points P_i as indicated below.

$$m_1 = 6, \qquad m_2 = 5, \qquad m_3 = 1, \qquad m_4 = 4$$

$$P_1 = (1, -2), \qquad P_2 = (3, 4), \qquad P_3 = (-3, -7), \qquad P_4 = (6, -1)$$

Find the moments M_x and M_y and the center of mass of the system.

Center of Mass of a Lamina

Consider a lamina with uniform density that occupies a region R. The center of mass of R is the centroid of R. To locate the centroid we need to follow 3 physical principles:

1. The symmetry principle: if R is symmetric about a line l the centroid of R lies on l.
2. Moments should be defined so that if the entire mass of the region is concentrated at the centroid, then its moments remain unchanged.
3. The moment of the union of 2 non-overlapping regions should be the sum of the moments of the individual regions.

Let $R = \{(x, y) : a \leq x \leq b,\ 0 \leq y \leq f(x)\}$ where f is a continuous function on $[a, b]$. R has uniform density ρ.

$$M_y = \rho \int_a^b x f(x)\, dx,$$

$$M_x = \frac{\rho}{2} \int_a^b [f(x)]^2\ dx.$$

If A denotes the area of R, then the center of mass of R is located at $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{1}{A} \int_a^b x f(x)\, dx,$$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)]^2\ dx.$$

Recall that $A = \displaystyle\int_a^b f(x)\, dx$