November 30, 2016

## Announcements

- This week 9.1 (Introduction to Differential Equations), 9.3 (Separable Equations)
- Homework # 9A (Center of Mass) & 9B (Separable differential equations) Due tonight, Wednesday, November 30, 11:00pm
- Homework # 10A & 10B (both on Differential Equations) due Wednesday, December 7 at 11:00 pm
- Printout and bring the Worksheet "DiffEQ.pdf" with you tomorrow, Thursday December 1 for TA sections

## Today

- Finish Problem from Monday
- Differential Equations
  - Mixing Problems
  - Radioactive Decay

**Problem 2 :** Find the solution of the differential equation

$$xy' + y = y^2, \qquad y(1) = -1.$$

# Mixing Problems (e.g.: a chemical dissolved in water)

Let $r_{in}$ and $r_{out}$ be the **rates** (in liters per minute) at which the liquid is entering and leaving the container respectively.

Let $C_{in}$ and $C_{out}$ be the **concentrations** of chemicals entering and leaving the container respectively.

Our independent variable is time, $t$, and the unknown (or dependent variable) is:
$y(t) =$ total amount of chemical in the container at time $t$.

Assuming that the container is always well mixed, we have

$$C_{out} = \frac{y(t)}{V(t)}$$

where $V(t)$ is the volume of liquid in the container at time $t$.

The differential equation is then

$$\begin{aligned} \frac{dy}{dt} &= C_{in} r_{in} - C_{out} r_{out} \\ &= C_{in} r_{in} - \frac{y(t)}{V(t)} r_{out} \end{aligned}$$

Note: If $r_{in} = r_{out}$ then the volume is constant. Otherwise

$$V(t) = V_0 + (r_{in} - r_{out})t.$$

**9.3 Problem: Warm up** A tank contains 1000L of pure water.

1. Brine that contains 0.05 kg of salt per liter enters the tank at a rate of 5 L/min.

The solution is kept thoroughly mixed and drains from the tank at a rate of 5 L/min. How much salt is in the tank:

- after $t$ minutes?
- after one hour?

**9.3 Problem:** A tank contains 1000L of pure water.

Brine that contains 0.05 kg of salt per liter enters the tank at a rate of 5 L/min.

Brine that contains 0.04 kg of salt per liter enters the tank at a rate of 10 L/min.

The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank:

- after $t$ minutes?
- after one hour?
- What happens in the long run (i.e. as $t \to \infty$)?