Announcements

- This week: 9.1 (Introduction to Differential Equations), 9.3 (Separable Equations)
- Homework # 10A & 10B (both on Differential Equations) due Wednesday, December 7 at 11:00 pm
- Bring a WiFi enabled device (phone, tablet or laptop) with you to class on Monday
- Bring ReviewOne.pdf and ReviewTwo.pdf to your TA sections on Tuesday, Dec. 6
- Final Exam: Saturday, December 10 from 1:30 – 4:20 pm (Bring your student ID)
  - Rooms: 125 A in KNE 110 and 125 B in KNE 210
  - Do sample Finals: See Math 125 Materials page
  - Send me requests for challenging Final problems to discuss in class.

Today

- Differential Equations
  - Radioactive Decay
  - Coffee Consumption
  - Newton's Law of Heating and Cooling

Last class: A tank contains 1000L of pure water. Brine that contains 0.05 kg of salt per liter enters the tank at a rate of 5 L/min and brine that contains 0.04 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank after t minutes?

We found the solution

$$\boxed{y(t) = \frac{130}{3}(1 - e^{-\frac{3t}{200}})}$$

and observed that

$$\lim_{t \to \infty} y(t) = \frac{130}{3}.$$

Given the rates at which they enter the tank, it makes sense that the long term amount should reflect 1/3 the concentration of the first brine mixture and two thirds concentration of the second brine mixture. Thus we would expect the amount of salt in the tank to approach

$$\frac{1}{3}(0.05)(1000) + \frac{2}{3}(0.04)(1000) = \frac{1}{3}(50) + \frac{2}{3}(40) = \frac{130}{3}.$$

Exponential growth and decay

The solution of the initial value problem

$$\frac{dy}{dt} = ky, \qquad y(0) = y_0$$

is given by

$$y(t) = y_0\, e^{kt} \tag{1}$$

Radioactive decay

Radioactive substances decay by spontaneously emitting radiation. If $m(t)$ is the mass remaining from an initial mass m_0 of substance after time t, then it has been found experimentally that

$$-\frac{1}{m}\frac{dm}{dt}$$

is constant. Thus

$$\frac{dm}{dt} = k\,m$$

where k is a **negative** constant. Equation (1) leads to the solution

$$m(t) = m_0\,e^{kt}$$

The **half life** is the time required for half of any given mass to decay, i.e. if T is the half life then

$$m(T) = \frac{m_0}{2} = m_0\,e^{kT} \qquad \Longrightarrow \qquad T = -\frac{\ln 2}{k} \quad \textit{or} \quad k = -\frac{\ln 2}{T}.$$

Problem: Bismuth 210 has a half-life of 5.0 days.

1. A sample originally has mass 800 mg. Find a formula for the mass remaining after t days.
2. Find the mass remaining after 30 days.
3. When is the mass reduced to 1 mg?

Coffee Consumption

Evidence shows that caffeine in the blood is metabolized at a rate proportional to the amount present.

UW researchers find that typically the amount of caffeine in the blood is reduced by $1/2$ in 1 hour.

A typical espresso (from the HUB) has approximately 50 mg of caffeine.

Problem: Suppose you have an espresso at time $t = 0$ (just before your calculus lecture).

(a) At time t how much caffeine is in your blood?

(b) How much caffeine is in your blood 5 hours later (when you go to the Math Study Center)?

Newton's Law of Cooling

Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large.
Let $T(t)$ be the temperature of the object at time t, let T_s be the temperature of the surroundings. Newton's Law of Cooling formulated as a differential equation states

$$\frac{dT}{dt} = k(T - T_s),$$

where k is a negative constant. If $T(0) = T_0$ then

$$T(t) = T_s + (T_0 - T_s)e^{kt}.$$

Problem: A thermometer is taken from a room where the temperature is 20°C to the outdoors, where the temperature is 5°C. After one minute the thermometer reads 12°C.

- What will the reading on the thermometer be after one more minute?
- When will the thermometer read 6°C?