## Announcements

- Assigned reading for the week sections 5.3, 5.4 and 5.5.
- Solutions to the Worksheets will be available soon after the Thursdays sections end. Go to the Math 125 Materials Website and click on "Outline 1" to find the link to the "Worksheet 1 Solutions".
- Homework #1 (125 HW 1ABC, all 3 parts) should be completed by tonight. Due Wednesday, October 5, 11:00pm.
- Quiz #1 (taken from HW #1ABC) on Tuesday, October 4 in TA sections.
- Print out and bring the second worksheet "Antiderivatives and Areas" with you to your section Thursday
- I have created a Catalyst GoPost Discussion Board for the class: https://catalyst.uw.edu/gopost/board/pollack/43156/

## Today

- 5.2 The Definite Integral: definition and properties.

# Riemann sums

When solving the area problem we encountered, **Riemann sums**, which are expressions of the form

$$\sum_{i=1}^{n} f(x_i^*)\Delta x = f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x.$$

where $x_i^*$ is any point in the $i^{th}$-subinterval.

We also **defined the Area** by the limits of Riemann sums as the number of rectangles tends to infinity, i.e.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} [f(x_1^*)\Delta x + f(x_2^*)\Delta x \cdots + f(x_n^*)\Delta x].$$

# The Definite Integral

## Definition

*Let $f$ be a continuous function defined on the interval $[a, b]$. Divide the interval $[a, b]$ into $n$-subintervals of equal width*

$$\Delta x = \frac{b-a}{n}.$$

*Let*

$$x_0 = a,\ x_n = b,\ x_{i+1} = x_i + \Delta x$$

*Let $x_i^*$ be a sample point in the $i^{th}$-subinterval $[x_{i-1}, x_i]$. The* **definite integral** *of $f$ from $a$ to $b$ is*

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

# Properties of Sums

If $\lambda$ is any constant and $n$ any positive integer then for any numbers $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ we have

$$\sum_{i=1}^{n} \lambda = n\lambda \tag{1}$$

$$\sum_{i=1}^{n} \lambda a_i = \lambda \sum_{i=1}^{n} a_i \tag{2}$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \tag{3}$$

$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i \tag{4}$$

# Properties of the definite integral

If $c$ is an point in $[a, b]$ and $\lambda$ is a constant then

- $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$
- $$\int_a^a f(x)\,dx = 0$$
- $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$
- $$\int_a^b \lambda\,dx = \lambda(b-a)$$
- $$\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$
- $$\int_a^b \lambda f(x)\,dx = \lambda \int_a^b f(x)\,dx$$

# Comparison properties for the integral

- If $f(x) \geq 0$ for $x$ in $[a, b]$, then

$$\int_a^b f(x)\,dx \geq 0$$

- If $f(x) \geq g(x)$ for $x$ in $[a, b]$, then

$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

- If $m \leq f(x) \leq M$ for $x$ in $[a, b]$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$