Announcements

- Assigned reading for the week sections 5.3, 5.4 and 5.5.
- Homework #1 (125 HW 1ABC, all 3 parts) must be completed by tonight, Wednesday, October 5, 11:00pm.
- Print out and bring the second worksheet "Antiderivatives and Areas" with you to your section Thursday
- Homework #2 (125 HW 2ABC, all 3 parts) should be completed by next Monday night, October 10. Due Wednesday, October 12, 11:00pm.
- Quiz #2 (taken from HW #2) on Tuesday, October 11 in TA sections.

Today

- 5.2 The Definite Integral: Application of comparison properties from Friday.
- 5.3 The Fundamental Theorem of Calculus
  - What is the relationship between differentiation and integration?
  - The Fundamental Theorem of Calculus
  - Some applications

Comparison properties for the integral

- If $f(x) \geq 0$ for x in $[a, b]$, then
$$\int_a^b f(x)\,dx \geq 0$$
- If $f(x) \geq g(x)$ for x in $[a, b]$, then
$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$
- If $m \leq f(x) \leq M$ for x in $[a, b]$, then
$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$

The Fundamental Theorem of Calculus

Let f be a continuous function on $[a, b]$.

1. The function defined by

$$g(x) = \int_a^x f(t)\,dt$$

   is continuous on $[a, b]$ and differentiable on (a, b). Moreover

$$g'(x) = f(x).$$

2. If F is any antiderivative of f, i.e $F' = f$ in (a, b) then

$$\int_a^b f(t)\,dt = F(b) - F(a).$$

Example

Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}.$$

Solution:

Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}.$$

Step 1: The first observation is that if the equality holds then the derivatives must also be equal:

$$\frac{d}{dx}(6 + \int_a^x \frac{f(t)}{t^2}\,dt) = \frac{d}{dx}(2\sqrt{x}).$$

By Part 1 of the Fundamental Theorem of Calculus, and an application of the power rule for derivatives, this gives us

$$\frac{f(x)}{x^2} = x^{-\frac{1}{2}}.$$

Solving for $f(x)$ we then get

$$f(x) = x^{3/2}.$$

Step 2: We may now use Part 2 of the Fundamental Theorem of Calculus to find a.
The LHS of the original equality with $f(t) = t^{3/2}$ is given by

$$\begin{aligned} 6 + \int_a^x \frac{f(t)}{t^2}\,dt &= 6 + \int_a^x \frac{t^{3/2}}{t^2}\,dt \\ &= 6 + \int_a^x t^{-1/2}\,dt \\ &= 6 + 2\sqrt{x} - 2\sqrt{a} \end{aligned}$$

(since $2\sqrt{t}$ is an antiderivative of $t^{-1/2}$).
Setting this equal to the RHS of the original equality we get

$$6 + 2\sqrt{x} - 2\sqrt{a} = 2\sqrt{x}.$$

This gives us

$$3 - \sqrt{a} = 0$$

from which we conclude that $a = 9$.