October 7, 2016

Announcements

- Assigned reading for the week sections 5.3, 5.4 and 5.5.
- Homework #2 (125 HW 2ABC, all 3 parts) should be completed by next Monday night, October 10. Due Wednesday, October 12, 11:00pm.
- Quiz #2 (taken from HW #2) on Tuesday, October 11 in TA sections.
- A number of people in each of the sections struggled with completing yesterday's Worksheet. Be sure to review the solutions to the Worksheet which are available here (or get there from the Math 125 Material page):
  http://www.math.washington.edu/˜ m125/Worksheets/sol2.pdf

Today

- 5.3 The Fundamental Theorem of Calculus
- 5.4: Indefinite Integrals & Net Change Theorem

The Fundamental Theorem of Calculus

Let f be a continuous function on $[a, b]$.

1. The function defined by

$$g(x) = \int_a^x f(t)\,dt$$

   is continuous on $[a, b]$ and differentiable on (a, b). Moreover

$$g'(x) = f(x).$$

2. If F is any antiderivative of f, i.e $F' = f$ in (a, b) then

$$\int_a^b f(t)\,dt = F(b) - F(a).$$

The Indefinite Intregral

Let f be a continuous function on an interval.

Definition

The indefinite integral of f is the most general antiderivative of f and it is denoted by

$$\int f(x)dx.$$

In particular the notation

$$F(x) = \int f(x)dx \quad \text{means} \quad F'(x) = f(x).$$

Notation

Let f be a continuous function on the interval $[a, b]$.

- $\displaystyle\int_a^b f(x)\,dx$ is a number

- $\displaystyle\int_a^x f(t)\,dt$ is an antiderivative of f whose value at a is 0

- $\displaystyle\int f(x)\,dx$ is the most general antiderivative of f

Let f be a continuous function on $[a, b]$. Let F be an antiderivative of f.

- $$\int_a^b f(x)\,dx = F(b) - F(a)$$
- $$\int_a^x f(t)\,dt = F(x) - F(a)$$
- $$\int f(x)\,dx = F(x) + C$$

Table of indefinite integrals

- $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ and $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

- $\displaystyle\int e^x\,dx = e^x + C$

- $\displaystyle\int \sin x\,dx = -\cos x + C$ and $\displaystyle\int \cos x\,dx = \sin x + C$

- $\displaystyle\int \sec^2 x\,dx = \tan x + C$ and $\displaystyle\int \frac{\sin x}{\cos^2 x}\,dx = \sec x + C$

- $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan x + C$ and $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

The net change rule

The integral of the rate of change is the net change,

$$\int_a^b F'(t)\,dt = F(b) - F(a).$$

Example: A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week.
What does

$$100 + \int_0^{15} n'(t)\,dt$$

represent?

Example

If an object moves along a straight line with position $s(t)$ its velocity is $v(t) = s'(t)$ and

$$\begin{aligned}\int_{t_1}^{t_2} v(t)\,dt &= s(t_2) - s(t_1)\\ &= \text{net change of position}\\ &= \text{displacement}\\ \int_{t_1}^{t_2} |v(t)|\,dt &= \text{total distance traveled}\end{aligned}$$

Displacement vs total distance traveled: Suppose

$$v(t) = 3t - 5 \qquad \text{on } 0 \le t \le 3.$$

(a) Find the displacement

(b) Find the total distance traveled between $t = 0$ and $t = 3$.