October 10, 2016

Announcements

- Assigned reading for the week sections 6.1, 6.2 and 6.3.
- Homework #2 (125 HW 2ABC, all 3 parts) should be completed by tonight Monday, October 10. Due Wednesday, October 12, 11:00pm.
- Quiz #2 (taken from HW #2) Tomorrow (Tuesday) October 11 in TA sections.
- Print out and bring the third worksheet: "Area between curves" with you to your section Thursday

Today

- Recap: Net Change Theorem: Displacement vs Distance Traveled
- Quick Review: The chain rule (3.4).
- 5.5: The Substitution Rule

Example

If an object moves along a straight line with position $s(t)$ its velocity is $v(t) = s'(t)$ and

$$\begin{aligned}\int_{t_1}^{t_2} v(t)\,dt &= s(t_2) - s(t_1) \\ &= \text{net change of position} \\ &= \text{displacement} \\ \int_{t_1}^{t_2} |v(t)|\,dt &= \text{total distance traveled}\end{aligned}$$

Displacement vs total distance traveled: Suppose

$$v(t) = 3t - 5 \qquad \text{on } 0 \leq t \leq 3.$$

(a) Find the displacement

(b) Find the total distance traveled between $t = 0$ and $t = 3$.

Displacement vs total distance traveled

Suppose

$$v(t) = 3t - 5 \qquad \text{on } 0 \le t \le 3.$$

(a) The displacement is given by

$$\begin{aligned} s(3) - s(0) &= \int_0^3 v(t)\,dt \\ &= \int_0^3 (3t-5)\,dt \\ &= (3\frac{t^2}{2} - 5t)\Big|_0^3 = \frac{27}{2} - 15 = -\frac{3}{2}. \end{aligned}$$

(b) The total distance traveled between $t = 0$ and $t = 3$ is given by

$$\int_0^3 |v(t)|\, dt \;=\; \int_0^3 |3t - 5|\, dt.$$

Since $3t - 5 = 0$ when $t = 5/3$ we see that this is

$$\begin{aligned}
\int_0^3 |v(t)|\, dt &= \int_0^{5/3} |3t - 5|\, dt + \int_{5/3}^3 |3t - 5|\, dt \\
&= \int_0^{5/3} (5 - 3t)\, dt + \int_{5/3}^3 (3t - 5)\, dt \\
&= \left(5t - 3\frac{t^2}{2}\right)\Big|_0^{5/3} + \left(3\frac{t^2}{2} - 5t\right)\Big|_{5/3}^3 \\
&= 5\left(\frac{5}{3}\right) - \frac{3}{2}\left(\frac{5}{3}\right)^2 + \frac{27}{2} - 15 - \frac{3}{2}\left(\frac{5}{3}\right)^2 + \frac{25}{3} = \frac{41}{6}.
\end{aligned}$$

The chain rule

If f and g are both differentiable and $F = f \circ g$ is the composition function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

The substitution rule

If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$