Announcements

- Assigned reading for the week sections 6.1, 6.2 and 6.3.
- Homework #2 (125 HW 2ABC, all 3 parts) Due tonight, Wednesday, October 12, 11:00pm.
- Print out and bring the third worksheet: "Area between curves" with you to your section Thursday
- Homework #3 (125 HW 3ABC, all 3 parts) Due Wednesday, October 19, 11:00pm (complete before section on Tuesday 10/18)
- Midterm #1 approaching fast: Thursday, October 20
  (More info on Friday).

Today

- Definite Integrals via Substitution
- 6.1: Areas between Curves

Definite integrals via Substitution

Recall from last class, using Substitution, we showed that

$$\int x^2\sqrt{1+x^3}\,dx = \frac{2}{9}\left(1+x^3\right)^{3/2} + C$$

Using this and the FTC II we can compute a definite intergal

$$\begin{aligned}\int_0^2 x^2\sqrt{1+x^3}\,dx &= \left.\frac{2}{9}\left(1+x^3\right)^{3/2}\right|_0^2 \\ &= \frac{2}{9}(9)^{3/2} - \frac{2}{9}(1)^{3/2} \\ &= \frac{54}{9} - \frac{2}{9} = \boxed{\frac{52}{9}}.\end{aligned}$$

Is there a way that we can avoid this 2-step process?

Definite integrals via Substitution II

We used substitutions $u = 1 + x^3$ and and $du = 3x^2\,dx$ to evaluate the integral. With the definite integral we have to be careful with the limits of integration:

$$
\begin{aligned}
\int_0^2 x^2\sqrt{1+x^3}\,dx &= \frac{1}{3}\int_1^9 \sqrt{u}\,du \\
&= \left.\frac{2}{9}u^{3/2}\right|_1^9 \\
&= \frac{2}{9}(9^{3/2} - 1^{3/2}) = \frac{2}{9}(27-1) = \boxed{\frac{52}{9}}.
\end{aligned}
$$

We changed the limits of integration because in the first integral they indicate the values $x = 0$ and $x = 2$. After the substitution $u = 1 + x^3$ these correspond to $u = 1$ and $u = 9$.

The substitution rule for definite integrals

If $u = g(x)$ is a differentiable function on $[a, b]$ and f is continuous function on the range of g, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

The problem of computing the area between two curves

Problem: Consider the region S that lies between 2 curves $y = f(x)$ and $y = g(x)$ and between the vertical lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a, b]$. Compute its area.

Areas between Curves

The area A of the region bounded by the curves $y = f(x)$ and $y = g(x)$ and between the vertical lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a, b]$ is

$$A = \int_a^b [f(x) - g(x)]\, dx$$

Exercise 1

a) Find the area of region bounded by the graphs of the curves $y = 0$ and $y = x^2 - 4$ between $x = -2$ and $x = 2$.

b) Find the area of region bounded by the graphs of the curves $y = 0$ and $y = x^2 - 4$ between $x = -3$ and $x = 3$.

Areas between Curves II

The area A of the region bounded by the curves $y = f(x)$ and $y = g(x)$ and between the vertical lines $x = a$ and $x = b$, where f and g are continuous functions in $[a, b]$ is

$$A = \int_a^b |f(x) - g(x)| \, dx$$

More Examples

2) Find the area of region bounded by the graphs of the curves $y = x^2$ and $y = x^4$.