Announcements

- Assigned reading/plan for the week sections 6.3, Midterm review, 6.4.
- Homework #3 (125 HW 3ABC, all 3 parts) Due Wednesday, October 19, 11:00pm (complete before section on Tuesday 10/18)
- Worksheet "Solids of Revolution and the Astroid" Tuesday (no quiz, MT review)
- Midterm #1 Thursday, October 20 in your sections
  - Covers all material in sections 4.9, 5.1 - 5.5 and 6.1 - 6.3
  - One 8.5 x 11, 2-sided, handwritten note sheet (use prep as a study aid!)
  - The only calculator which may be used is the Ti-30x IIS.
  - GIVE EXACT ANSWERS. You will lose points if you do not give the exact answer to a problem and instead provide a decimal approximation
  - Do sample midterms from Math 125 Materials webpage (click on "MIDTERM #1 Achive")
  - CLUE Review: Tuesday, Oct. 18, 8:00-9:30pm (note time) in MGH 241

Today

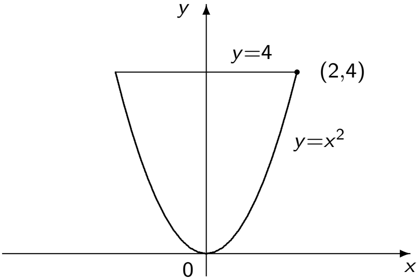
- 6.2 Volume (slicing method)
- 6.3: Volumes by Cylindrical Shells

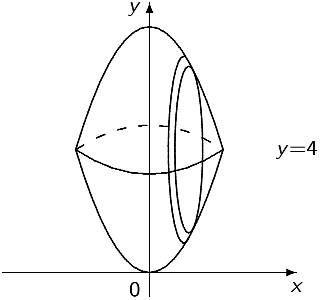
Volumes by Slicing

Definition (Slicing method): Let S be a solid that lies between $x = a$ and $x = b$. If the area of the cross section of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \int_a^b A(x)\, dx$$

Example 2 - from Friday: Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 4$ about $y = 4$.





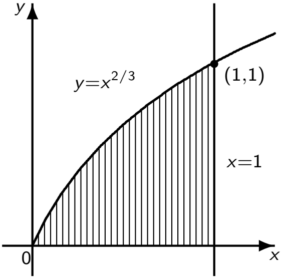
Solution: The slice at x, $-2 \leq x \leq 2$, perpendicular to the x-axis, is a disk of radius $r(x)$ and area $A(x) = \pi r(x)^2$. From the picture of the region we can see that

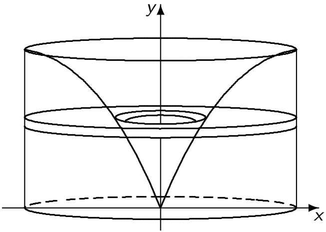
$$r(x) = 4 - x^2 \qquad \text{and therefore} \qquad A(x) = \pi(4 - x^2)^2.$$

So using the slicing method to find the volume we see that

$$\begin{aligned} V &= \int_{-2}^{2} A(x)\, dx = \int_{-2}^{2} \pi(4 - x^2)^2\, dx \\ &= 2\pi \int_{0}^{2} (16 - 8x^2 + x^4)\, dx \\ &= 2\pi \left[16x - \frac{8}{3}x^3 + \frac{x^5}{5} \right] \Bigg|_0^2 \\ &= 2\pi \left[32 - \frac{64}{3} + \frac{32}{5} \right] \\ &= \boxed{\frac{512}{15}\pi}. \end{aligned}$$

Example 1 - Today: Find the volume of the solid obtained by rotating the region bounded by $y = x^{2/3}$, $x = 1$ and $y = 0$ about y-axis.



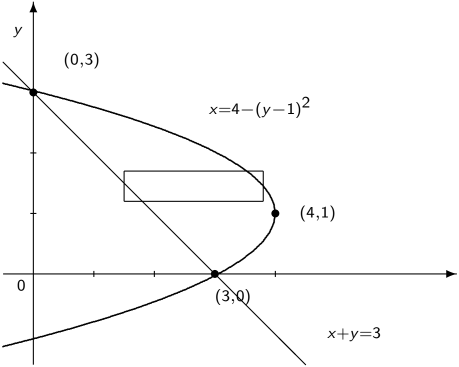


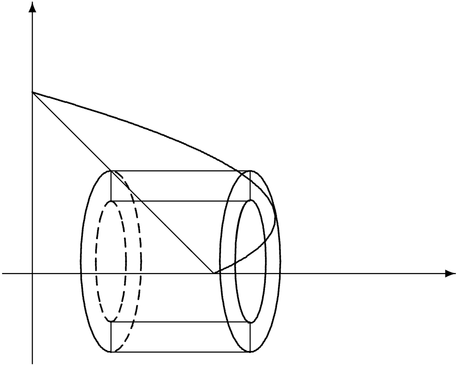
Computing Volume using Cylindrical Shells

The volume of the solid obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b (with $0 \leq a < b$), is

$$V = \int_a^b 2\pi x f(x)\, dx$$

Example 2: Find the volume of the solid obtained by rotating the region bounded by $y + x = 3$ and $x = 4 - (y-1)^2$ about the x-axis.





Solution to Example 2: Since the region is rotated about the x-axis to obtain the solid of revolution, we consider shells whose axis is horizontal.

To compute the infinitesimal volume dV of a shell, we need to find the width, the circumference and the height of the shell.

- Width $= dy$
- Circumference $= 2\pi y$
  (y is the distance between the shell and the axis of rotation, this is the radius of the shell)
- Height $= [4 - (y-1)^2] - [3 - y]$
  (the height extends from the curve $x = 4 - (y-1)^2$ and the line $x = 3 - y$)

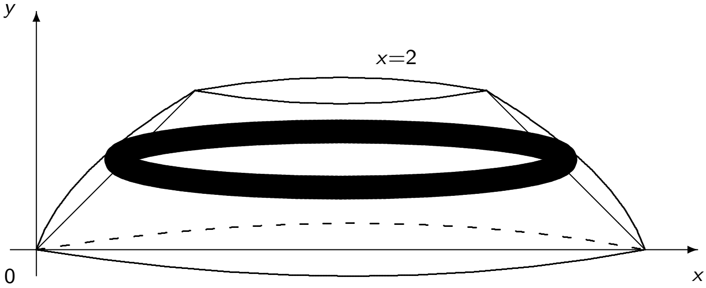
Thus

$$\begin{aligned} dV &= 2\pi y[4-(y-1)^2-(3-y)]\,dy \\ &= 2\pi y(3y-y^2)\,dy. \end{aligned}$$

The smallest shell has radius 0 and the largest has radius 3. These are our bounds of integration.

$$\begin{aligned} V &= \int_0^3 2\pi y(3y-y^2)dy \\ &= 2\pi(y^3-\frac{y^4}{4})\Big|_0^3 \\ &= 2\pi(27-\frac{81}{4}) = \frac{27}{2}\pi. \end{aligned}$$

Example 3: Find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about $x = 2$.



Solution: Since the region is rotated about a line parallel to the y-axis to obtain the solid of revolution, we slice perpendicular to the y-axis.

The slices begin at $y = 0$ and end at $y = 1$, these will therefore be our limits of integration.

Solving for x as a function of y, we see that the curves are given by $x = y$ and $x = y^2$.

The inner radius at y extends from the line $x = y$ to the vertical line $x = 2$. Therefore the inner radius is

$$r(y) = 2 - y.$$

The outer radius at y extends from the curve $x = y^2$ to the vertical line $x = 2$. Therefore the outer radius is

$$R(y) = 2 - y^2.$$

The area of the washer at slice y is then

$$\begin{aligned} A(y) &= \pi(R(y))^2 - \pi(r(y))^2 \\ &= \pi[(2-y^2)^2 - (2-y)^2] \\ &= \pi[y^4 - 5y^2 + 4y] \end{aligned}$$

Putting this all together we see that the volume of the solid of revolution is

$$\begin{aligned} V &= \int_0^1 A(y)\,dy \\ &= \int_0^1 \pi[y^4 - 5y^2 + 4y]\,dy \\ &= \pi\,[\frac{1}{5}y^5 - \frac{5}{3}y^3 + 2y^2]\Big|_0^1 \\ &= \pi[\frac{1}{5} - \frac{5}{3} + 2] \\ &= \frac{8\pi}{15}. \end{aligned}$$