

Solution (§6.2 #16): Since the region is rotated about the y -axis to obtain the solid of revolution, we slice parallel to the y -axis.

The slices begin at $y = 0$ and end at $y = 1$, these will therefore be our limits of integration.

Solving for x as a function of y , we see that the curves are given by $x = y$ and $x = y^2$.

The inner radius at y extends from the line $x = y$ to the vertical line $x = 2$. Therefore the inner radius is

$$r(x) = 2 - y.$$

The outer radius at y extends from the curve $x = y^2$ to the vertical line $x = 2$. Therefore the outer radius is

$$R(x) = 2 - y^2.$$

The area of the washer at slice y is then

$$\begin{aligned} A(x) &= \pi(R(x))^2 - \pi(r(x))^2 \\ &= \pi[(2 - y^2)^2 - (2 - y)^2] \\ &= \pi[y^4 - 5y^2 + 4y] \end{aligned}$$

Putting this all together we see that the volume of the solid of revolution is

$$\begin{aligned} V &= \int_0^1 A(y) dy \\ &= \int_0^1 \pi[y^4 - 5y^2 + 4y] dy \\ &= \pi\left[\frac{1}{5}y^5 - \frac{5}{3}y^3 + 2y^2\right]\Big|_0^1 \\ &= \pi\left[\frac{1}{5} - \frac{5}{3} + 2\right] \\ &= \frac{8\pi}{15}. \end{aligned}$$