March 31, 2006

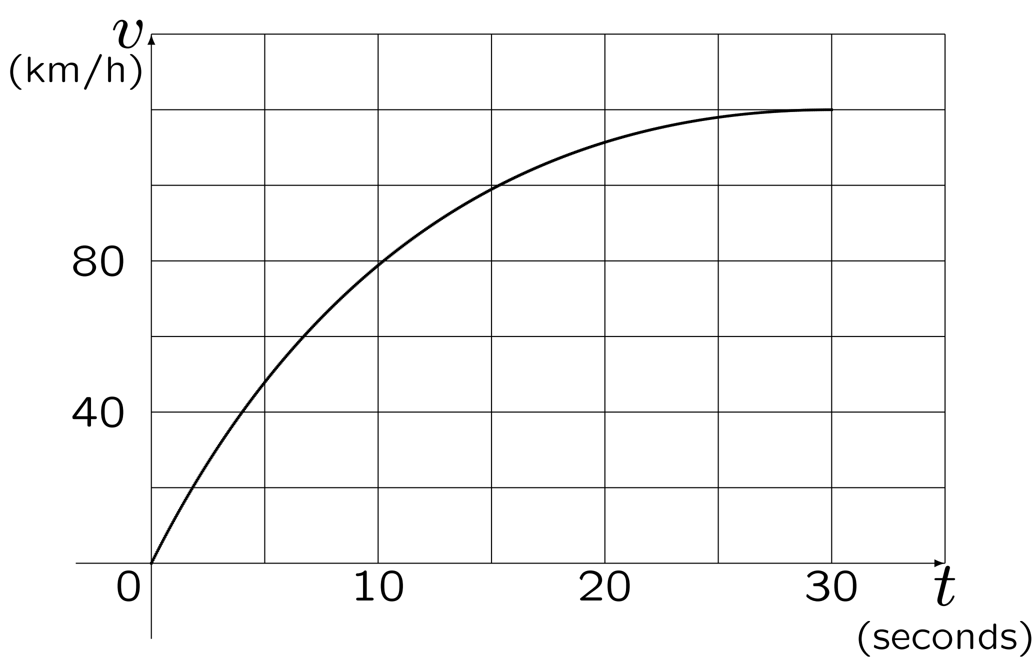
Announcements

- This Week: §4.10, 5.1 and 5.2
- Homework #1 (Week 1 Problems)
  Due Tuesday, April 4
  (Covers §4.10, 5.1 and 5.2; see web for assignment)
- Quiz # 1 Tuesday, April 4
  (Covers §4.10, 5.1 and 5.2, Week 1 Problems).

Today

- (Finish velocity problem from) §5.1: Areas and Distances
- §5.2 The Definite Integral: definition and properties.

The velocity graph of a car accelerating from rest to a speed of 120 km/h over a period of 30 seconds is shown. Estimate the distance traveled during this period.



When solving the area problem we encountered, **Riemann sums**, which are expressions of the form

$$\sum_{i=1}^{n} f(x_i^*)\Delta x =$$

$$f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x.$$

We also define the Area by their limits, i.e.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x =$$

$$\lim_{n\to\infty} [f(x_1^*)\Delta x + f(x_2^*)\Delta x \cdots + f(x_n^*)\Delta x].$$

Definition: Let f be a continuous function defined on the interval $[a, b]$. Divide the interval $[a, b]$ into n-subintervals of equal width

$$\Delta x = \frac{b-a}{n}.$$

Let

$$x_0 = a, \; x_n = b, \; x_{i+1} = x_i + \Delta x$$

Let $x_i^* \in [x_{i-1}, x_i]$ be sample points. The **definite integral of f from a to b** is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

Properties of sums

- $\sum_{i=1}^{n} \lambda = n\lambda$
- $\sum_{i=1}^{n} \lambda a_i = \lambda \sum_{i=1}^{n} a_i$
- $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$
- $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

Properties of the definite integral

If $c \in [a, b]$ and λ is a constant then

- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^b \lambda\,dx = \lambda(b - a)$
- $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
- $\int_a^b \lambda f(x)\,dx = \lambda \int_a^b f(x)\,dx$

Comparison properties for the integral

- If $f(x) \geq 0$ for $x \in [a, b]$, then

$$\int_a^b f(x)\,dx \geq 0$$

- If $f(x) \geq g(x)$ for $x \in [a, b]$, then

$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

- If $m \leq f(x) \leq M$ for for $x \in [a, b]$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$