May 3, 2006

Announcements

- This Week: §7.4, 7.5 and 7.7
- Homework #6 (Week 6 Problems) Due Tuesday, May 8 (Covers §7.4, 7.5 and 7.7; see web page for problems)
- Quiz #6 Tuesday, May 8 (Covers §7.4, 7.5 and 7.7, Week 5 Problems).

Today

- Mid Quarter Student Feedback
- Finish §7.4 Integration of Rational Functions by Partial Fractions
- §7.5 Strategy for Integration

Table of indefinite integrals

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int \frac{1}{x} \, dx = \ln |x| + C$
- $\int e^x \, dx = e^x + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \sec^2 x \, dx = \tan x + C$
- $\int \frac{1}{\sin^2 x} \, dx = -\frac{\cos x}{\sin x} + C$
- $\int \sec x \, dx = \frac{1}{2} \ln \left(\frac{1 + \sin x}{1 - \sin x} \right) + C$
- $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \arctan \left(\frac{x}{a} \right) + C$
- $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \left(\frac{x}{a} \right) + C$

Steps to evaluate an integral

1. Simplify the integrand

2. Does substitution work?

3. Does integration by parts work?

4. Classify the integrand:

   - Trigonometric function
   - Rational function
   - Contains radicals

5. Take a second look: either substitution or integration by parts must work (maybe after manipulation of the integrand).

Integration strategy

You might want to follow these steps when evaluating an integral. Be prepare to abandon an attempt and proceed with another. Do not get discouraged. It takes lots of practice to master all the techniques.

Steps:

1. Simplify the integrand. Keep in mind that it is easier to integrate a sum than a product.

2. Attempt an easy substitution. General form

$$\int f(g(x))g'(x)\,dx \quad \text{substitution :} \quad u = g(x).$$

Examples:

- $\int x^2\sqrt{x^3+1}\,dx$,

  substitution $u = x^3 + 1$.

- $\int (\sin x)e^{\cos x}\,dx$,

  substitution $u = \cos x$.

- $\int \dfrac{\ln x}{x}\,dx$,

  substitution $u = \ln x$.

3. Attempt a trigonometric substitution. General forms:

   - Expression $\sqrt{a^2 - x^2}$,
     substitution $x = a \sin\theta$.

   - Expression $a^2 + x^2$,
     substitution $x = a \tan\theta$.

   - Expression $\sqrt{x^2 - a^2}$,
     substitution $x = a \sec\theta$.

   Examples:

   - $\displaystyle\int \frac{1}{\sqrt{49 - 9x^2}}\, dx$, substitution $x = \frac{7}{3}\sin\theta$.

   - $\displaystyle\int \frac{1}{(x^2 + 25)^4}\, dx$, substitution $x = 5\tan\theta$.

   - $\displaystyle\int (x^2 - 16)^{5/2}\, dx$,
     substitution $x = 4\sec\theta$.

4. Attempt a rationalizing substitution. Although there is no general form in this case, the idea is to substitute the terms that bothers you the most in terms of being able to evaluate the integral.

   Examples:

   - $\int e^{x^{1/3}}\,dx$,
     substitution $u = x^{1/3}$, i.e. $u^3 = x$.
   - $\int \dfrac{e^{3x}}{e^{2x} + 5e^x + 4}\,dx$,
     substitution $u = e^x$.
   - $\int \dfrac{\sin x}{(\cos^2 x + 4)(\cos x + 3)}\,dx$,
     substitution $u = \cos x$.

5. Rational functions: use long division (if necessary) then partial fractions.

   Recall that if $f(x) = \frac{R(x)}{Q(x)}$ where $\deg R < \deg Q$, we mostly focused on the cases where Q is a product of linear factors (including repeated factors). We also need to be able to deal with irreducible quadratic factors in the denominator.

6. Integration by parts. General form

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

or

$$\int u\,dv = uv - \int v\,du.$$

Examples:

- $\int x^n \cos x\,dx.$
- $\int x^n \ln(x+3)\,dx$
- $\int x^n e^{4x}\,dx.$

7. If everything above fails, take a second look. Remember that either substitution or integration by parts must work maybe after manipulation of the integrand (for example: check to see if a trigonometric identity can be applied)