Homework consists of four parts: the written assignment W1-W4 which counts towards the final grade for the course, the extra credit problems which are for those who are interested in taking the Putnam competition and anyone in search of an opportunity to boost his or her grade, the presentation assignment (which will not be handed in) and (occasionally) a reading assignment.

Please don't get discouraged if you cannot immediately solve all of the problems, especially the presentation problems and the extra credit problems. The instructors are more than happy to discuss any of the problems by e-mail and in person - in particular, during Monday office hours - before the assignment is due. Hints are given upon request.

This homework has no reading assignment.

Written assignment (4 problems).

Writing Problem 1. There are 2015 distinct positive integers placed in a circle. Is it possible that the ratio of any two consecutive numbers (the largest to smallest) is a prime? What if we replace 2015 with 2014?

Writing Problem 2. For which positive integers N there exist N distinct positive integers such that the sum of all of them is divisible by each one of them?

Writing Problem 3. You are given 2016 integer numbers. Show that you can choose several of them such that their sum is divisible by 2016.

Writing Problem 4. A grasshopper jumps along a straight line. Each jump can go either left or right. His first jump is 1 cm, the second is 2 cm, etc. Can he get to his initial position in 2015 jumps? What about 2017?

Extra Credit Problem 1. Define

$$q(n) = \left\lfloor \frac{n}{\lfloor \sqrt{n} \rfloor} \right\rfloor .$$

Determine (with proof) all positive integers n such that

(i) $q(n) < q(n+1)$,

(ii) $q(n) = q(n+1)$,

(iii) $q(n) > q(n+1)$.

Presentation assignment (4 problems).

Presentation Problem 1. The sequence of Fibonacci numbers is defined by

$$F_1 = F_2 = 1, \quad F_{n+1} = F_n + F_{n-1} \quad \text{if } n \geq 2.$$

(a) Prove that any positive integer can be represented as a sum of several different Fibonacci numbers.
(b) Prove that F_n is divisible by 3 if and only if n is divisible by 4.

Presentation Problem 2. The n-dimensional space is colored with n colors such that every point in the space is assigned a color. Show that there exist two points of the same color exactly a mile from each other.

Presentation Problem 3. A 1×1 square with the upper left corner cut out from it is called a *tromino* (see below).

(a) What is the largest number of squares on an 8×8 chessboard which can be colored purple, so that any tromino on the board has at least one square which is not colored purple?

(b) What is the smallest number of squares on an 8×8 chessboard which can be colored purple, so that any tromino on the board has at least one square which is colored purple?

Presentation Problem 4. Prove that there exists an integer whose decimal presentation consists only of 1's which is divisible by 2017.