We strongly recommend that you read Chapter 8 (you may know all or some of it depending on your background in geometry).

**Written assignment (5 problems).**

**Writing Problem 1.** Let $ABC$ be a triangle, and $A'$, $B'$ and $C'$ be points on $BC$, $AC$ and $AB$ respectively such that $AA'$, $BB'$ and $CC'$ are concurrent at the point $G$. Prove that

$$\frac{GA'}{AA'} + \frac{GB'}{BB'} + \frac{GC'}{CC'} = 1$$

**Writing Problem 2.** The triangle formed by joining midpoints of the sides of a given triangle is called the medial triangle.

1. Prove that the medial triangle and the original triangle have the same centroid
2. Prove that the orthocenter of the medial triangle is the center of the circumscribed circle of the original triangle.

**Writing Problem 3.** A cone has circular base of radius 1, and its vertex is at height 3 directly above the center of the circle. A cube has four vertices on the base and four on the sloping sides. What is the side-length of the cube?

**Writing Problem 4.** Let $\triangle ABC$ be an acute triangle. Prove that

$$h_a > \frac{1}{2}(AC + AB - BC)$$

where $h_a$ is the length of the altitude on the side $BC$.

**Writing Problem 5.** Let $C$ be a circle in the $xy$-plane with center on the $y$-axis and passing through points $A = (0, a)$, $B = (0, b)$ with $0 < a < b$. Let $O$ be the origin $(0, 0)$, let $P$ be any point on the circle, and let $Q$ be the intersection of the line through $P$ and $A$ with the $x$-axis. Prove that

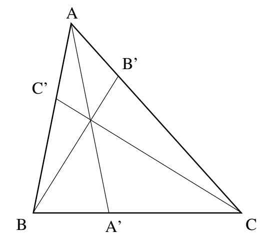
$$\angle BQP \cong \angle BOP.$$

<u>**EC.**</u> In a triangle $ABC$ choose point $A_1$ on side $BC$, point $B_1$ on side $CA$, and point $C_1$ on side $AB$ in such a way that the three segments $AA_1$, $BB_1$, and $CC_1$ intersect in one point $P$. Prove that $P$ is the centroid of triangle $ABC$ if and only if $P$ is the centroid of triangle $A_1B_1C_1$.

**Presentation assignment (4 problems).**

**Presentation Problem 1.** Prove *Ceva's theorem*: Let $ABC$ be a triangle, and $A'$, $B'$ and $C'$ be points on the interiors of $BC$, $AC$ and $AB$ respectively. Then $AA'$, $BB'$ and $CC'$ are concurrent if and only if $\frac{AC' \cdot BA' \cdot CB'}{C'B \cdot A'C \cdot B'A} = 1$.

*Note:* for the forward direction you **must** use an argument different from the one presented in the book. For example, an argument with weights. This is reviewed in 8.4.21 in Zeitz. Each part of this problem will be worth 3 presentation points.




**Presentation Problem 2.** Prove Pick's formula: Let $P$ be a convex polygon with vertices on the integer lattice. Let $i$ be the number of the interior integer points of $P$, and $b$ be the number of integer points on the sides (boundary) of the polygon. Prove that the area of $P$ equals $i + \frac{b}{2} - 1$.

**Presentation Problem 3.** Let $S$ be a circle with the center $(a, b)$ where $a, b$ are both irrational numbers. Show that $S$ cannot contain more than 2 points with rational coordinates.

**Presentation Problem 4.** Find

$$\arctan\frac{1}{2} + \arctan\frac{1}{8} + \ldots + \arctan\frac{1}{2n^2} \quad .$$

Arctan($t$) for $t \geq 0$ denotes the number $\theta$ in the interval $0 \leq \theta < \pi/2$ with $\tan(\theta) = t$. You might find the following formula useful: $\arctan x - \arctan y = \arctan\frac{x-y}{1+xy}$.