MATH 380 **Practice Final** **Autumn 2016**

Due December 1, 2016

NAME: SIGNATURE:

Instructions: there are 6 problems on the final. You need to solve (completely and impeccably) four of them to get 100%. Partial credit will be given on all 6 but sparringly. One complete solution is worth more than several partial ones. Please check your solutions very carefully and justify all your claims.

Problem	Number of points	Points obtained
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total		

Problem 1. Find all real solutions (a, b, c, d) to $a + b + c = d$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{d}$.

Problem 2. Does there exist a polynomial $p(x)$ with real coefficients (other than $p(x) \equiv 0$) such that

$$xp(x-1) = (x+1)p(x)$$

for all $x \in \mathbb{R}$?

Problem 3. Evaluate the infinite product

$$\prod_{n=2}^{\infty} \frac{n^3 - 1}{n^3 + 1}$$

(Hint: write down the first few terms explicitly.)

Problem 4. King Meanie has had it with legal fights over inheritance in the backward, male-centric society which he rules. He decides to pass a law under which every family is forbidden to have anymore children after a boy is born to them. (Giving birth to a girl or a boy is equally likely, and one can ignore multiple births, as they happen very rarely.)

What will happen to the gender balance in that land? (I.e., what will be the expected female-to-male ratio in children?)

Problem 5.

a) Show that it is not possible to use a piece of wire to construct the frame of a cube, without cutting the wire (no overlaps allowed).

b) What is the minimal number of cuts necessary in order to construct the frame of a cube?

Problem 6. In how many ways can a $2 \times n$ rectangle be tiled with 2×1 dominos? Give the answer in terms of a well-known sequence of numbers (and prove that it is correct).