Math 380a, Fall 2018 **HOMEWORK #6** *due Tue, November 6*

Reading: Section 5.5 in [Zeitz].
Civic duty: don't forget to vote!

Written assignment (5 problems).

Writing Problem 1. Let $a_1, \ldots, a_n$ be positive numbers. Show that

$$(a_1 + \cdots + a_n)\left(\frac{1}{a_1} + \cdots + \frac{1}{a_n}\right) \geq n^2$$

Writing Problem 2. a. Prove AM-GM for four numbers.

b. Prove AM-GM for three numbers. Hint: Use part a).

c. Prove AM-GM in general. Use the strategy developed in parts a), b).

Writing Problem 3. Let $a_1, \ldots, a_n, b_1, \ldots, b_n$ be positive numbers. Show that $\frac{a_1+\ldots+a_n}{b_1+\ldots+b_n}$ is between the smallest and the largest elements in the set $\left\{\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots, \frac{a_n}{b_n}\right\}$.

Writing Problem 4. Suppose that $a_1, a_2, \ldots, a_n$ with $n \geq 2$ are real numbers larger than -1, and moreover assume that all a_j's have the same sign. Show that

$$(1 + a_1)(1 + a_2)\ldots(1 + a_n) > 1 + a_1 + a_2 + \ldots + a_n.$$

Extra Credit Problem 1. Let $a, b, c, d \geq 0$. Prove that $\frac{1}{a} + \frac{1}{b} + \frac{4}{c} + \frac{16}{d} \geq \frac{64}{a+b+c+d}$.

Presentation assignment (5 problems).

Presentation Problem 1. Let $n \geq 2$. Prove that $\frac{1}{2^3} + \frac{1}{3^3} + \ldots + \frac{1}{n^3} < \frac{1}{2}$.

Presentation Problem 2. Let $a_1, \ldots, a_n$ be a sequence of positive numbers and let $b_1, \ldots, b_n$ be any permutation of the first sequence. Show that

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} + \ldots + \frac{a_n}{b_n} \geq n$$

Presentation Problem 3. Let x, y, z be positive real numbers such that $xyz = 1$. What is the minimal value of

$$\frac{x}{y+z} + \frac{y}{x+z} + \frac{z}{y+x}?$$

Hint: Read the book!

Presentation Problem 4. Let a be a real number and n a positive integer, with $a > 1$. Show that

$$a^n - 1 > n\left(a^{\frac{n+1}{2}} - a^{\frac{n-1}{2}}\right).$$

Presentation Problem 5. For which integer n is $1/n$ the closest to $\sqrt{1000000} - \sqrt{999999}$?