Math 380a, Fall 2018 ***HOMEWORK #7*** ***due Tue, November 13***

Reading: Chapter 9 [Zeitz].

Written assignment (4 problems).

Writing Problem 1. Define the sequence (a_n) by $a_0 = \alpha$ and

$$a_{n+1} = 5a_n - a_n^2$$

for $n \geq 1$. Explain/prove whether the sequence converges or not for $\alpha = 5$. What about for $\alpha = -1$?

Writing Problem 2. Let x be a real number such that $|x| < 1$. Compute

$$\lim_{n\to\infty} \prod_{i=1}^{n} \left(1 + x^{2^i}\right).$$

Writing Problem 3. Let $x_0 = 1$, and $x_{n+1} = x_n + 10^{-10^{x_n}}$ for all $n \geq 1$. What can we say about $\lim\limits_{n\to\infty} x_n$?

Writing Problem 4. Given $a > 1$, find $\lim\limits_{x\to\infty} \left(\frac{1}{x} \cdot \frac{a^x-1}{a-1}\right)^{\frac{1}{x}}$.

Extra Credit Problem 1. Evaluate

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \left(\lfloor \frac{2n}{k} \rfloor - 2\lfloor \frac{n}{k} \rfloor \right) \quad .$$

Presentation assignment (5 problems).

Presentation Problem 1. Find the limit

$$\lim_{n\to\infty} \left(\prod_{k=1}^{n} \left(1 + \frac{k}{n}\right) \right)^{\frac{1}{n}} \quad .$$

Presentation Problem 2. For a fixed, positive integer k, the n^{th} derivative of $\frac{1}{x^k-1}$ has the form $\frac{P_n(x)}{(x^k-1)^{n+1}}$, where $P_n(x)$ is a polynomial. Find $P_n(1)$.

Presentation Problem 3. Evaluate $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{n}{n^2+j^2}$.

Presentation Problem 4. Let $f(x)$ be a positive valued function over the reals such that $f'(x) > f(x)$ for all x. For what k must there exists N such that $f(x) > e^{kx}$ for $x > N$?

Presentation Problem 5. For any real number α, we denote by $\{\alpha\}$ the fractional part of α. Consider the sequence $a_n = \{n\sqrt{2}\}$, $n \geq 1$.

1. Show that the sequence a_n contains a subsequence that converges to 0 .
2. Show that the set a_n, $n \geq 1$, is *dense* in the interval $[0, 1]$. (Recall that a subset S is called *dense* if for any real number $x \in [0, 1]$, any open interval centered at x contains a number from S.)