

Presentation assignment (6 problems)

Presentation Problem 1. From where he stands, one step toward the cliff would send a drunken man over the edge. He takes random steps on a line, either toward or away from the cliff. At any step his probability of taking a step away is p (the probability of taking a step toward the cliff is $1 - p$).

What is the probability that he falls off the cliff after precisely k steps? Write the probability that he eventually falls off the cliff as an infinite sum. (Hint: look up Catalan numbers.)

Presentation Problem 2. Suppose n numbers are chosen uniformly and randomly from the interval $[0, 1]$. What is the expected value of the largest of them?

Presentation Problem 3. Suppose that x and y are chosen at random (with uniform density) and independently from the interval $(0, 1)$. What is the probability that the closest integer to x/y is even?

Presentation Problem 4. Suppose you are given a set of n *biased* coins, such that the probability that the m th coin will land on “heads” is $\frac{1}{2^{m+1}}$. If you flip all n coins independently exactly once, what is the probability that you get an odd number of heads? (Hint: try to use the same method as in class.)

Presentation Problem 5. Jonah and Julia are tossing a coin. Jonah made 2018 tosses, and Julia – 2019. What is the probability that Julia got more heads than Jonah?

Presentation Problem 6. Given a value $p \in (0, 1)$, devise a fair coin-tossing game which, with probability 1, ends after a finite number of tosses, and which you can win with probability p .