We strongly recommend that you read Chapter 8 (you may know all or some of it depending on your background in geometry).

**Written assignment (5 problems).**

**Writing Problem 1.** Let $C$ be a circle in the $xy$-plane with center on the $y$-axis and passing through points $A = (0,a)$, $B = (0,b)$ with $0 < a < b$. Let $O$ be the origin $(0,0)$, let $P$ be any point on the circle, and let $Q$ be the intersection of the line through $P$ and $A$ with the $x$-axis. Prove that

$$\angle BQP \cong \angle BOP.$$ 

**Writing Problem 2.** Let $ABCD$ be a quadrilateral and denote by $K,L,M,N$ the midpoints of the sides $AB,BC,CD,AD$ respectively, and by $P,Q$ the midpoints of the diagonals $AC$ and $BD$. Prove that $PLQN$ and $PKQM$ are parallelograms with the same center.

**Writing Problem 3.** Let $\triangle ABC$ be an acute triangle. Prove that

$$h_a > \frac{1}{2}(AC + AB - BC)$$

where $h_a$ is the length of the altitude on the side $BC$.

**Writing Problem 4.** Let $ABC$ be a right triangle with $\angle C = 90^\circ$ and $\angle A = \theta$. Let $X$ be a point on the hypotenuse $AB$ such that $AX = AX = 1$. Let $Y$ be a point on $BC$ such that $\angle CXY = \theta$. Let $Z$ be a point on the hypotenuse $AB$ where it meets the perpendicular to $BC$ at the point $Y$. Evaluate

$$\lim_{\theta \to 0} YZ.$$

**Writing Problem 5.** Let $ABC$ be a triangle, and $A', B'$ and $C'$ be points on $BC$, $AC$ and $AB$ respectively such that $AA'$, $BB'$ and $CC'$ are concurrent at the point $G$. Prove that

$$\frac{GA'}{AA'} + \frac{GB'}{BB'} + \frac{GC'}{CC'} = 1$$

**Presentation assignment (4 problems).**

**Presentation Problem 1.** Prove Ceva’s theorem: Let $ABC$ be a triangle, and $A'$, $B'$ and $C'$ be points on the interiors of $BC$, $AC$ and $AB$ respectively. Then $AA'$, $BB'$ and $CC'$ are concurrent if and only if $\frac{AC'.BA'.CB'}{C'B.A'C.BA} = 1$. 
Note: for the forward direction you must use an argument different from the one presented in the book. For example, an argument with weights. This is reviewed in 8.4.21 in Zeitz. Each part of this problem will be worth 3 presentation points.

Presentation Problem 2. Let $R$ be the radius of the circumscribed circle, and $r$ be the radius of the inscribed circle of a triangle $ABC$. Prove that

$$R \geq 2r.$$ 

Presentation Problem 3. Prove Pick's formula: Let $P$ be a convex polygon with vertices on the integer lattice. Let $i$ be the number of the interior lattice points of $P$, and $b$ be the number of lattice points on the sides (boundary) of the polygon. Prove that the area of $P$ equals $i + \frac{b}{2} - 1$.

Presentation Problem 4. Find

$$\arctan \frac{1}{2} + \arctan \frac{1}{8} + \ldots + \arctan \frac{1}{2n^2}.$$ 

Arctan($t$) for $t \geq 0$ denotes the number $\theta$ in the interval $0 \leq \theta < \pi/2$ with $\tan(\theta) = t$. You might find the following formula useful: $\arctan x - \arctan y = \arctan \frac{x-y}{1+xy}$. 