Writing Problem 1. Let $a_1, \ldots, a_n$ be positive numbers. Show that
\[
(a_1 + \cdots + a_n)\left(\frac{1}{a_1} + \cdots + \frac{1}{a_n}\right) \geq n^2
\]

Writing Problem 2. Prove AM-GM for four numbers.

Writing Problem 3. Let $a_1, \ldots, a_n, b_1, \ldots, b_n$ be positive numbers. Show that $\frac{a_1 + \cdots + a_n}{b_1 + \cdots + b_n}$ is between the smallest and the largest elements in the set $\left\{\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots, \frac{a_n}{b_n}\right\}$.

Writing Problem 4. Suppose that $a_1, a_2, \ldots, a_n$ with $n \geq 2$ are real numbers larger than $-1$, and moreover assume that all $a_j$'s have the same sign. Show that
\[
(1 + a_1)(1 + a_2)\ldots(1 + a_n) \geq 1 + a_1 + a_2 + \ldots + a_n.
\]

Extra Credit Problem 1. Find the largest number in the sequence
\[
a_n = \frac{1000^n}{n!},
\]
where $n$ is a positive integer, $n \geq 1$.


Presentation Problem 2. Let $a_1, \ldots, a_n$ be a sequence of positive numbers and let $b_1, \ldots, b_n$ be any permutation of the first sequence. Show that
\[
\frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n} \geq n
\]

Presentation Problem 3. Let $a$ be a real number and $n$ a positive integer, with $a > 1$. Show that
\[
a^n - 1 > n\left(a^{\frac{n+1}{2}} - a^{\frac{n-1}{2}}\right).
\]

Presentation Problem 4. For which integer $n$ is $1/n$ the closest to $\sqrt{1000000} - \sqrt{999999}$?