Math 380a HOMEWORK #9 due Tue, December 2, 2014
Written assignment (3 problems).

W1. From where he stands, one step toward the cliff would send a drunken man over the
edge. He takes random steps on a line, either toward or away from the cliff. At any step
his probability of taking a step away is $p$ (the probability of taking a step toward the cliff is $1 - p$).

a) Let $\pi$ be the probability that he will eventually fall off the cliff, if his drunken walk
starts one step away from it. Let $\hat{\pi}$ be the probability that he will eventually fall off
the cliff if he starts two steps away from it. Express $\hat{\pi}$ in terms of $\pi$.

b) Find a quadratic equation satisfied by $\pi$, by considering what may happen after the
first step, together with part a).

c) The quadratic equation in b) has two possible solutions, depending on whether you
choose the $+$ or the $-$ in the above. Assume also that $\pi$ is a continuous function of $p$.
Which one of the two solutions is $\pi$?

W2. Let $a$ be the diameter of a small coin. Given a chessboard $C$ made of squares of
size $d \times d$ and with delimiting lines of negligible width, throw the coin “at random” on the
chessboard. By “at random” we mean that the center of the circular coin can fall uniformly
on any point on the chessboard (the margin of the coin may be outside the chessboard, if
the center is close to the edge). What is the probability that the coin will fall entirely within
one of the individual $d \times d$ squares?

W3. Suppose $n$ numbers are chosen uniformly and randomly from the interval $[0, 1]$. What
is the expected value of the largest of them?

Presentation assignment (4 problems, in addition to the Practice Final.)

P1. Let $p_n$ be the probability that two numbers selected at random, independently and
uniformly, from $\{1, 2, \ldots, n\}$ sum up to a square (e.g., $1 + 3 = 2^2$). Find $\lim_{n \to \infty} p_n \sqrt{n}$.

P2. Three players enter a room and a red or blue hat is placed on each player’s head. The
color of each hat is determined by an independent coin toss. Each of the players can see the
other players’ hats but not their own.

No communication (of any sort) is allowed, except for an initial strategy session before
the game begins. Once they have had a chance to look at the other hats, the players must
simultaneously guess the color of their own hats, or pass.

Find a strategy that has at least one person guessing, and no players guessing incorrectly,
with probability more than $1/2$. (For example, if one of them guessed and the others passed,
the guess would be right with probability $1/2$; we want you to do better.)
P3. Let $C$ be the circle of radius 1 centered at the origin. Choose a point $P$ at random on the circle, and a point $Q$ at random inside the circle. What is the probability that the rectangle with diagonal $PQ$ and sides parallel to the axes lies entirely in or on the circle?

P4. Ioana and Julia are tossing a coin. Ioana made 2013 tosses, and Julia—2014. What is the probability that Julia got more heads than Ioana?