## 1. Pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \quad\quad 1 + \tan^2 x = \sec^2 x$$

## 2. Sum-Difference formulas

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y} \quad\quad \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot x \pm \cot y}$$

$$\arctan(\alpha) \pm \arctan(\beta) = \arctan\left(\frac{\alpha \pm \beta}{1 \mp \alpha\beta}\right)$$

## 3. Double Angle formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

## 4. Half Angle formulas

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \quad\quad \cos^2 x = \frac{1 + \cos(2x)}{2} \quad\quad \tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

## 5. Sum to product formulas

$$\sin x + \sin y = 2 \sin\left(\tfrac{x+y}{2}\right)\cos\left(\tfrac{x-y}{2}\right) \quad\quad \sin x - \sin y = 2\cos\left(\tfrac{x+y}{2}\right)\sin\left(\tfrac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\tfrac{x+y}{2}\right)\cos\left(\tfrac{x-y}{2}\right) \quad\quad \cos x - \cos y = -2\sin\left(\tfrac{x+y}{2}\right)\sin\left(\tfrac{x-y}{2}\right)$$

## 6. Product to sums formulas

$$\sin x \sin y = \tfrac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \tfrac{1}{2}(\cos(x - y) + \cos(x + y))$$

$$\sin x \cos y = \tfrac{1}{2}(\sin(x + y) + \sin(x - y))$$

## 7. Another useful trick

$$A \sin x + B \cos x = \sqrt{A^2 + B^2} \sin(x + \alpha), \quad \text{where } \alpha = \arccos \frac{A}{\sqrt{A^2 + B^2}}$$