Quiz 1 Solutions

Problem 1. *Give an example of two* 3×3 *matrices that are not equivalent. Justify your answer.*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

These matrices are not equivalent because they are both in reduced echelon form but are not equal.

Problem 2. *Give an example of a matrix that is in echelon form but not in reduced echelon form.*

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Decide whether each of the following is true or false. Justify your answers.

Problem 3. *A homogeneous linear system with seven variables and five equations must have infinitely-many solutions.*

True. Since it is homogeneous, it is consistent (will have at least the trivial solution). So we can put the system into echelon form and since it has more variables than equations it must have free variables and hence infinitely many solutions.

Problem 4. *If two linear systems have infinitely-many solutions then they are equivalent.*

False. For example, consider the linear system given by the equation $x_1 + x_2 = 1$ and the system given by the equation $x_1 + x_2 = 0$. These both have infinitely many solutions but the solution sets are different. To see why the solutions sets are different, notice that $(0, 0)$ is a solution to one but not the other.

Problem 5. *If A, B and C are sets satisfying $A \subset B$ and $B \subset C$, then $A \subset C$.*

True. Suppose $x \in A$. Then $x \in B$ since $A \subset B$. But then since $B \subset C$ we also have $x \in C$. Therefore any element of A is also an element of C, and so $A \subset C$.

Problem 6. *If two matrices are in echelon form and are equivalent, then they must be the same matrix.*

False. Consider for example $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ *and* $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.