Annie's Survival Kit 3 - Math 324

Six questions for the price of three this week!

1. (10 points) (a) (7 points) Switch the order of integration of $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}+1}^{1} 1\,dz\,dx\,dy$ to $dr\,d\theta\,dz$.

   (b) (3 points) Knowing that $\int\int\int_R 1dV$ calculates the volume of a region R, solve the previous triple integral without doing any calculations.

2. (10 points) Switch the order of integration of $\int_0^{\pi}\int_0^1\int_{-1}^1 zr^3\,dz\,dr\,d\theta$ to $dy\,dx\,dz$.

3. (10 points) Consider a solid cone of height $\sqrt{3}$ with a 120° vertex angle. Its density at point P is equal to the distance from P to the central axis of the cone. Set up the integrals for the mass of the cone using cylindrical coordinates in two different orders: $dz\,dr\,d\theta$ and $dr\,d\theta\,dz$. Do not evaluate those integrals.

   Hints:

   - Choose and place the coordinate system to get the easiest integral possible.
   - The mass of a solid region R with density δ is $\int\int\int_R \delta\,dV$.
   - If the cone has a $\frac{2\pi}{3}$ vertex angle (the angle between its sides), what is the slope of its sides? How does the slope fit into the equation for a cone?

4. (10 points) Set up a triple integral to find the volume of the region bounded by $z \le x^2+y^2$, $x^2+y^2 \le 3$ and $z \ge 0$ using **spherical coordinates**. (Recall that volume is $\int\int\int_R 1\,dV$.) **Do not evaluate.**

5. (10 points) Switch $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_2^3 zr^4\,dz\,dr\,d\theta + \int_0^{2\pi}\int_{\sqrt{3}}^2\int_2^{\sqrt{4-r^2}+2} zr^4\,dz\,dr\,d\theta$ to spherical coordinates.

6. (10 points) Find the area of the ellipse $(2x+5y-7)^2+(3x-7y+1)^2 \le 1$.