RAINWATER SEMINAR

Conformal growth rates and spectral geometry on unimodular random graphs

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1:30-3:30

For a unimodular random graph (G, x), we consider deformations of its intrinsic path metric by a (random) weighting of its vertices. This leads to the notion of the conformal growth exponent of (G, ρ), which is the best asymptotic degree of volume growth of balls that can be achieved by such a weighting. Under moment conditions on the degree of the root, the conformal growth exponent of a unimodular random graph coincides with its almost sure spectral dimension (whenever the latter exists).
We show that distributional limits (a la Benjamini and Schramm) of finite graphs that can be sphere-packed in R^d have conformal growth exponent at most d, and thus the connection to spectral dimension yields d-dimensional lower bounds on the heat kernel.
In two dimensions, one obtains more precise information. If (G, x) has a property we call "quadratic conformal growth," then the following holds: If the degree of the root is uniformly bounded almost surely, then G is almost surely recurrent. Since H-minor-free graphs have this property, limits of such graphs are almost surely recurrent, affirming a conjecture of Benjamini and Schramm (2001). For the special case of planar graphs, this gives a proof of the Benjamini-Schramm Recurrence Theorem that does not proceed via the analysis of circle packings.
These methods extend to models with unbounded degrees, giving new proofs of almost sure recurrence for the well-studied uniform infinite planar triangulation (UIPT) and quadrangulation (UIPQ). They yield quantitative bounds on the heat kernel, spectral measure, and speed of the random walk. For instance, we are able to recover the known $n^{1/3}$ speed bound for UIPQ using only planarity and quartic volume growth.