RAINWATER SEMINAR

Rectifiability of harmonic measure

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C-401

1:30-3:30

We are going to consider one, two and three phase problems for harmonic measure.
1. In a recent multi-authored paper by J. Azzam, S. Hofmann, J.-M. Martell, S. Mayboroda, M. Mourgoglou, X. Tolsa and myself the following result is proved:
For arbitrary open set in any $\mathbb{R}^d$ such that its boundary carries harmonic measure w and for any Borel set E, such that $H^{d-1}(E) < \infty$, if harmonic measure $w|E$ is absolutely continuos with respect to $H^{d-1}|E$, then it is rectifiable.
This result solves a long standing conjecture of Chris Bishop and generalizes the result of N. Makarov obtained for d=2 and for simply connected open sets obtained in the early 80's.
2. For the two phase problem it turns out that id harmonic measures in two disjoint domains are absolutely continuous with respect to each other, then the interface of these two domains is rectifiable, and they are both absolutely continuous with respect to the "surface measure. In this generality (namely, without any assumption on domains whatsoever) this was recently proved by J. Azzam, M. Mourgoglou, X. Tolsa and myself.
3. The three phase problems claims that three disjoint domains cannot ever be mutually absolutely continuous. This was proved by Boris Tsirelson by purely probabilistic methods, but it turns out that there exists a simple potential theory approach (found by Xavier Tolsa and myself) that proves this result.
We will also give an overview of harmonic measure results of the last 30 years.