RAINWATER SEMINAR

Reversibility of the Loewner energy

Yilin Wang, ETH Zurich

January 17, 2017

C-401

1:30-3:30

Loewner introduced in 1923 a way to encode non-self-intersecting two-dimensional paths by a one-dimensional continuous driving function. This method is an important tool in the proof of Bieberbach conjecture and gives rise to a natural family of random curves, SLEs. We define the Loewner energy of a deterministic chord as the Dirichlet energy of its driving function. It is a priori defined for a directed chord from one boundary point A of a simply connected domain D to another boundary point B, and is conformally invariant. Using an interpretation of this energy as a large deviation rate function for SLE_κ as κ goes to 0, we show that the energy is reversible, i.e. it remains the same if the chord is viewed as going from B to A in D. In consequence, the Loewner energy measures how far does the chord differ from the hyperbolic geodesic.
The first part of the talk consists of a brief overview of the Loewner theory and the Loewner energy. I will try to convey an intuition on what the Loewner energy measures. In the second part, I will introduce SLE, large deviation principle and how are they related to the Loewner energy and its reversibility. I will outline the proof. If time allows, I will present the Loewner energy for loops on the Riemann sphere, and open questions related to it.