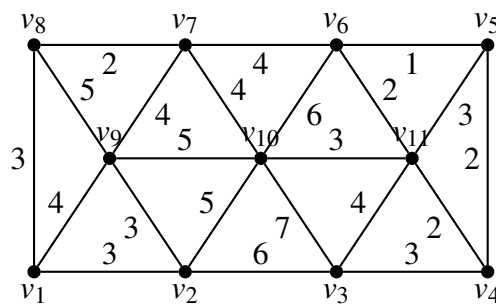Problem Set 1

514 - Networks and Combinatorial Optimization

Autumn 2022

Exercise 1.7 (10 pts)
Find, both with the Dijkstra-Prim algorithm and with Kruskal's algorithm, a spanning tree of minimum length in the graph in the figure below.

Exercise 1.9 (10 pts)
Let $G = (V, E)$ be a graph and let $\ell : E \to \mathbb{R}$ be a length function. Call a forest F *good* if $\ell(F') \geq \ell(F)$ for each forest satisfying $|F'| = |F|$.

Let F be a good forest and e be an edge not in F so that $F \cup \{e\}$ is a forest and such that (among all such e) $\ell(e)$ is as small as possible. Show that $F \cup \{e\}$ is good again.

Remark. Both exercises are taken verbatim from A. Schrijver's lecture notes.