CHAPTER 23

LAKE EUTROPHICATION MANAGEMENT: THE LAKE BALATON PROJECT

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Abstract

This is a brief overview of a collaborative effort of the Environment and Natural Resources, and the Adaptation and Optimization task forces at IIASA, to design stochastic optimization models for the management of lake eutrophication, and its use in a major case study (Lake Balaton). For further details, consult: Somlyódy [**5**],[**6**]; Somlyódy and van Straten [**8**]; Somlyódy and Wets [**9**]; Rockafellar and Wets [**2**]; and King [**1**].

Lake Balaton (Figure 23.1), one of the largest shallow lakes of the world, which is also the center of the most important recreational area in Hungary, has recently exhibited the unfavorable signs of artificial eutrophication. An impression of the major features of the lake-region system (including phosphorus sources and control alternatives) can be gained from Figure 23.1 (for details, see Somlyódy et al [**7**]; and Somlyódy and van Straten [**8**]). Four basins of different water quality can be distinguished in the lake (Figure 23.1) determined by the increasing volumetric nutrient load from east to west (the biologically available phosphorus load, BAP, is about ten times higher in Basin I than in Basin IV). The latter is associated to the asymmetric geometry of the system, namely the smallest western basin drains half of the total watershed, while only 5% of the catchment area belongs to the larger basin.

Based on observations for the period 1971–1982 the average deterioration of water quality of the entire lake is about 10% (in terms of Chlorophyll-a (Chl-a)). According to the OECD classification, the western part of the lake is in a (most advanced) hypertrophic state (which is the result of the large nutrient load), while the eastern portion of it is in an eutrophic stage.

The modeling approach to eutrophication and its management involved 4 major phases (Somlyódy [**5**]).

1. The description of the dynamics of the lake eutrophication process by a simulation model (LEM) which has two sets of inputs: controllable inputs (mainly artificial nutrient loads) and noncontrollable inputs (meteorological factors, such as temperature, solar radiation, wind, precipitation). The output of the model is the concentrations vector y of a number of water

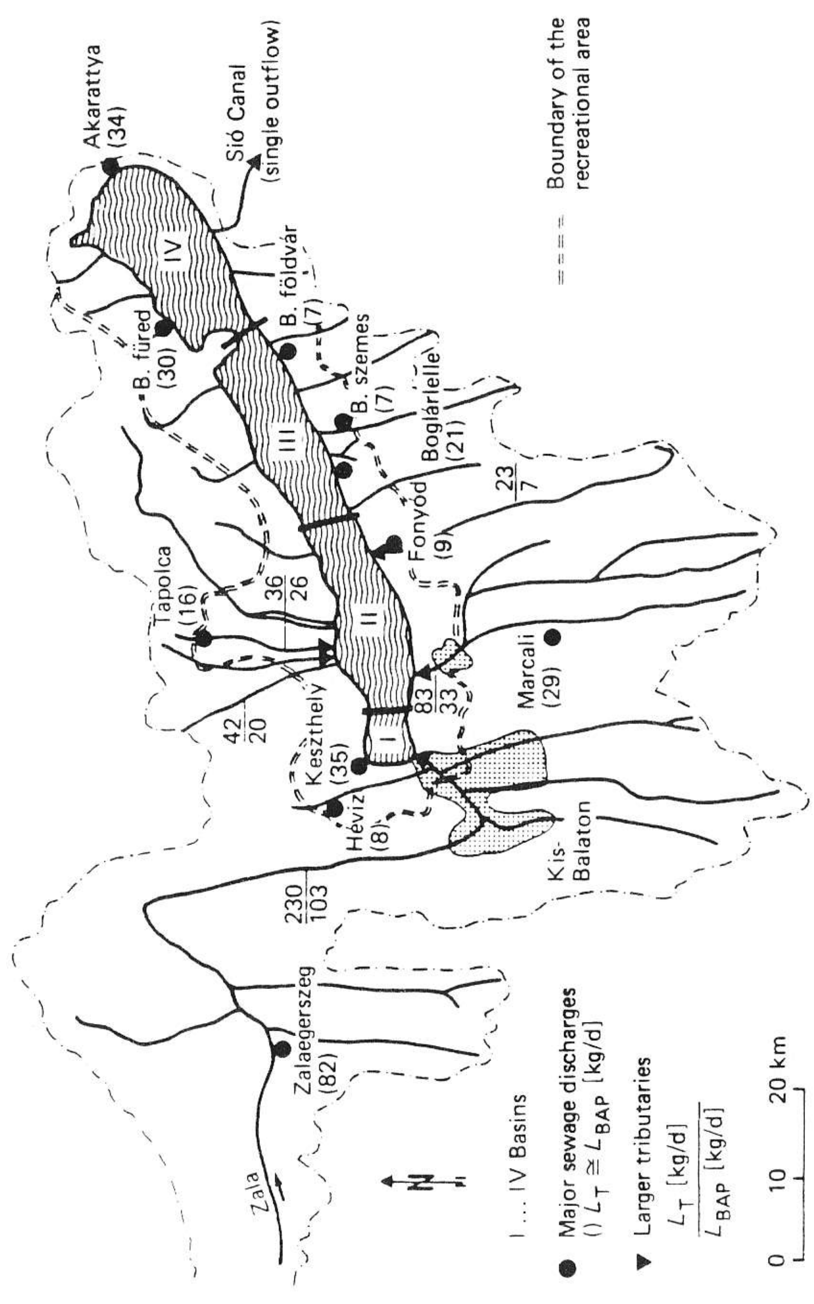

Figure 23.1 Major nutrient sources and control options.

quality components as a function of time (on a daily basis) t, and space $r : y(t, r)$. LEM is calibrated and validated by relying on historical data.

2. Derivation of stochastic inputs and the usage of LEM in a Monte Carlo fashion under systematically changed load conditions resulting in water quality as a stochastic variable: $\tilde{y}(t, r)$. Selection of the indicator for water quality management: for Lake Balaton the annual peak value of (Chl-a) was found to be appropriate. The use of (Chl-a)max as the indicator allows to eliminate time from the analysis on the level of management.
3. Derivation of the aggregated, stochastic load response model (LEMP) serving the indicator as a function of the load (for Lake Balaton a linear relationship was obtained). Design of a planning type nutrient load model (NLMP) and the incorporation of LEMP and NLMP in a management, optimization model (EMOM).
4. Validation. In the course of this procedure various simplifications and aggregations are made without a quantitative knowledge of the associated errors. Accordingly, the last step in the analysis is validation. That is, the LEM should be run with the "optimal" load scenario (found in the previous step), and the "accurate" and "approximate" solutions generated by the aggregated and nonaggregated versions of LEM can be compared.

The lakes' total P is in an average $315t/yr$ (the BAP load is $170t/yr$); but depending on the hydrologic regime it can reach $550t/yr$. 53% of the load L is carried by tributaries (30% of which is of sewage origin—indirect load, see e.g., the largest city of the region, Zalaegerszeg in Figure 23.1), 17% is associated to direct sewage discharges (the recipient is the lake). Atmospheric pollution is responsible for 8% of the lake's load and the rest comes from direct runoff (urban and agricultural). Tributary load increases from east to west, while the change in the direct sewage load goes in the opposite direction. The sewage contribution (direct and indirect loads) is 30% to P, while it is about 52% to the total biological available load (the load of agricultural origin can be estimated as 47% and 33%, respectively) suggesting the importance of sewage load from the viewpoint of the short term eutrophication control. Figure 23.1 indicates also the loads of sewage discharges and tributaries which were involved in the management optimization model. These cover about 85% of the nutrient load which we consider controllable on the short term (e.g. atmospheric pollution and direct runoff are excluded).

Control alternatives are sewage treatment (upgrading of the biological stage and introduction of P precipitation) and the establishment of prereservoirs as indicated in Figure 23.1 (see e.g. the Kis-Balaton reservoir system planned for a surface area of about 75 km^2).

The nutrient load model for Lake Balaton incorporates control variables associated with control options mentioned. Sewage load was considered deterministic, while tributary load was modeled by the simple relationship.

$$\mathbf{L} = (L_0 + a_1 \mathbf{Q} + \mathbf{L}_p)(\xi^- + \boldsymbol{\xi})$$

where L_0 is the base load (mainly of sewage origin), $\mathbf{Q}$ is the stream flow rate, $\mathbf{L}_p$ is the residual, and the variable $\boldsymbol{\xi}$ accounts for the influence of infrequent sampling (ξ^- is the lower bound). The most detailed data set including 25 years of continuous records for Q and 5 years of daily observations for the loads was available for the Zala River (Figure 23.1) draining half of the watershed and representing practically the total load of Basin I. For the Zala River $\mathbf{L}_p$ was found to have a normal distribution, while $\mathbf{Q}$ was approached by a lognormal distribution. Tributary load can be controlled by choosing the size of reservoirs (they generally consist of two parts, having separated impacts on dissolved and particulate loads, see Figure 23.1), while the L_0 component can be influenced by sewage treatment. As can be judged from the above equation, sewage treatment affects the expectation of the load, only, while reservoirs affects both expectation and variance (for details see Somlyódy [6]).

The planning type nutrient load model (NLMP) outlined briefly and the linear load response model (LEMP) lead to the affine relation (Somlyódy and Wets [9])

$$\mathbf{y}(x,\omega) = \mathbf{T}(\omega)x - \mathbf{h}(\omega)$$

where $y = (y_1, \ldots, y_4)$ are the water quality indicators in Basins 1,...,4, the random vector $\mathbf{h}$ incorporates all noncontrollable factors, the x-variables are the control variables and the linear transformation $T(\omega)x$ gives the effect on water quality of the measures taken to control the loads $\mathbf{L}$.

In the formulation of the eutrophication management optimization model (EMOM) the objective must be chosen so as to measure in the most realistic fashion possible the deviations of the indicators from the water quality goals. This led us to a stochastic program with recourse model with associated solution procedure developed by Rockafellar and Wets [3] and implemented by King [1]. We also used a linear programming model, see Somlyódy [6] and Somlyódy and Wets [9] (Section 6) that is based on expectation-variance considerations (for the water quality indicators). In the Lake Balaton case study the results for both this expectation-variance model and the stochastic programming model (5.11) lead to remarkably similar investment decisions. Subsequently, objective functions and results of the two models are briefly discussed.

1. The recourse formulation starts from the following considerations. The model should distinguish between situations that barely violate the desired water quality levels (γ_i, $i = 1, \ldots, N$) and those that deviate substantially from these norms. This suggests a formulation of our objective in terms of a penalization that would take into account the observed values of $(y_i(x,w) - \gamma_i)$ for $i = 1, \ldots, 4$.

We found that the following class of functions provided a flexible tool for the analysis of these factors. Let $\theta : R \to R_+$ be defined by

$$\theta(\tau) := \begin{cases} 0 & \text{if } \tau \le 0 \\ \frac{1}{2}\tau^2 & \text{if } 0 \le \tau \le 1 \\ \tau - \frac{1}{2} & \text{if } \tau \ge 1 \end{cases}$$

This is a piecewise linear-quadratic-linear function. The penalty functions (Ψ_i, $i = 1, \ldots, N$) are defined through:

$$\Psi_i(z_i) = q_i e_i \theta(e_i^{-1} z_i) \text{ for } = 1, \ldots, N,$$

where q_i and e_i are positive quantities that allow us to scale each function Ψ_i in terms of slopes and the range of its quadratic component. By varying the parameters e_i and q_i we are able to model a wide range of preference relationships and study the stability of the solution under perturbation of these scaling parameters.

The objective is thus to find a program that in the average minimizes the penalties associated with exceeding the desired concentration levels. This leads to the following formulation of the water quality management problem:

$$\begin{aligned}
&\text{find} \quad x \in R^n \text{ such that} \\
&\qquad 0 \le x_j \le r_j, \qquad j = 1, \ldots, n \\
&\qquad \sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad i = 1, \ldots, m_1 \\
&\qquad \sum_{j=1}^{n} t_{ij}(w) x_j - v_i(w) = h_i(w) \qquad i = 1, \ldots, m_2 \\
&\text{and } z = \sum_{j=1}^{n} \left(c_j x_j + \frac{d_j}{2r_j} x_j^2 \right) + E\{\sum_{i=1}^{m_2} q_i e_i \theta\left(e_i^{-1} v_i(w)\right)\} \text{ is minimized}
\end{aligned}$$

to which one refers as *a quadratic stochastic program with simple recourse*; here b_1 is the available budget that we handle as a parameter. For problems of this type, in fact with this application in mind, an algorithm is developed in Rockafellar and Wets [2], and Rockafellar and Wets [3], which relies on the properties of an associated dual problem. In particular it is shown that the following problem:

$$\begin{aligned}
&\text{find} \quad y \in R_+^m \text{ and } z(\cdot) : \Omega \to R^{m_2} \text{ measurable such that} \\
&\qquad 0 \le z_i(w) \le q_i, \qquad i = 1, \ldots, m_2 \\
&\qquad u_j = c_j - \sum_{i=1}^{m_1} a_{ij} y_i - E\{\sum_{i=1}^{m_2} z_i(w) t_{ij}(w)\}, \qquad j = 1, \ldots, n \\
&\text{and} \quad \sum_{i=1}^{m_1} y_i b_i - \sum_{i=1}^{m_2} E\{h_i(w) z_i(w) + \frac{e_i}{2q_i} z_i^2(w)\} \\
&\qquad - \sum_{j=1}^{n} r_j d_j \theta(d_j^{-1} u_j) \text{ is maximized ,}
\end{aligned}$$

is dual to the original problem, provided that for $i = 1, \ldots, m_2$, the e_i and q_i are positive (and that is the case here) and for $j = 1, \ldots, n$, the $d_j > 0$, which is taken care of by a natural perturbation of the objective.

An experimental version of this algorithm that relies on MINOS was implemented at IIASA by A. King (and is available through IIASA as part of a collection of codes for solving stochastic programs), see King [1]. It starts the procedure by solving the deterministic problem with expected values for the coefficients in h and T.

2. As a starting point for the construction of the expectation-variance model, we consider the following objective function:

$$\sum_{i=1}^{N} q_i E\{y_i(x,\cdot)-\gamma_i)_+^2\}$$

where, as earlier, $y_i(x,w)$ is the water quality indicator characterized by the selected indicator in basin i given the investment program x and the environmental conditions w, γ_i the goal set for basin i and q_i a weighting factor. The objective being quadratic in the area of interest, and the distribution functions $G_i(x,\cdot)$ of the $y_i(x,\cdot)$ not being too far from normal, one should be able to recapture the essence of the effect of this objective function on the decision process by considering just expectations and variances of the $y_i(x,\cdot)$. This observation, and the "soft" character of the management problem, suggest that we could substitute for the original objective

$$\sum_{i=1}^{N} q_i \left(E\{y_i(x,\cdot)-\bar{y}_{oi}\}+\theta\sigma(y_i(x,\cdot)-\bar{y}_{oi})\right)$$

where θ is a positive scalar (usually between 1 and 2.5), $\bar{y}_{oi}=E\{y_{oi}\}$ is the expected nominal state of basin i, and σ denotes standard deviation,

$$\sigma\left(y_i(x,\cdot)-\bar{y}_{oi}\right)=E\left\{\left(y_i(x,\cdot)-E\{y_i(x,\cdot)\}\right)^2\right\}^{\frac{1}{2}}.$$

Since for each $i=1,\ldots,N$, the y_i are affine (linear plus a constant term) with respect to x, the expression for

$$E\{y_i(x,\cdot)-\bar{y}_{oi}\}=\sum_{j=1}^{n}\mu_{ij}x_j+\mu_{io}$$

as a function of x is easy to obtain from the load equations. The μ_{ij} are the expectations of the coefficients of the x_j and the μ_{io} the expectation of the constant term. Unfortunately the same does not hold for the standard deviation $\sigma(y_i(x,\cdot)-\bar{y}_{oi})$. The nutrient-load model suggest that

$$\sigma\left(y_i(x,\cdot)-\bar{y}_{oi}\right)\sim\left(\sum_{\ell}\sigma_{i\ell}^2x_\ell^2\right)^{\frac{1}{2}}$$

where $\sigma_{i\ell}$ is the part of the standard deviation that can be influenced by the decision variable x_ℓ; for example, the standard deviation of the tributary load.

Cross terms are for all practical purposes irrelevant in this situations since the total load in basin i is essentially the sum of the loads generated by various sources that are independently controlled. This justifies using

$$\sum_{i=1}^{N} q_i\left[\left(\sum_{j=1}^{n}\mu_{ij}x_j\right)+\theta\left(\sum_{j=1}^{n}\sigma_{ij}^2x_j^2\right)^{\frac{1}{2}}\right]$$

as an objective for the optimization problem. This function is convex and differentiable on R_+^n except at $x=0$, and conceivably one could use a nonlinear programming package to solve the optimization problem:

$$\begin{aligned}&\text{find } x\in R^n \text{ such that}\\ &r_j^-\le x_j\le r_j^+ \qquad j=1,\ldots,n\\ &\sum_{j=1}^{n}a_{ij}x_j\le b_i \qquad i=1,\ldots,m\\ &\text{and } z=\sum_{i=1}^{N}q_i\left[\sum_{j=1}^{n}\mu_{ij}x_j+\theta\left(\sum_{j=1}^{n}\sigma_{ij}^2x_j^2\right)^{\frac{1}{2}}\right] \text{ is minimized.}\end{aligned}$$

One can go one step further in simplifying the problem to be solved, namely by replacing the term.

$$\left(\sum_{j=1}^{n}\sigma_{ij}^2x_j^2\right)^{\frac{1}{2}}$$

in the objective, by the linear (inner) approximation

$$\sum_{j=1}^{n}\sigma_{ij}x_j.$$

On each axis of R_+^n, no error is introduced by relying on this linear approximation; otherwise we are over-estimating the effect a certain combination of the $x_j's$ will have on the variance of the concentration levels. Thus, at a given budget level we shall have a tendency to start projects that affect more strongly the variance if we use the linear approximation, and this is actually what we observed in practice. Assuming the cost functions c_j are piecewise linear, we have to solve the *linear* program:

$$\begin{aligned}&\text{find } x\in R^n \text{ such that}\\ &r_j^-\le x_j\le r_j^+, \qquad j=1,\ldots,n\\ &\sum_{j=1}^{n}a_{ij}x_j\le b_i, \qquad i=1,\ldots,m_1\end{aligned}$$

$$\text{and } t = \sum_{i=1}^{N} q_i \sum_{j=1}^{n} (\mu_{ij} + \theta\sigma_{ij}) x_j \text{ is minimized.}$$

We refer to this problem as the *(linearized) expectation-variance model.*

We have given only a heuristic "justification" for the use of the expectation-variance model as a management tool. In Section 6 of Somlyódy and Wets [9], this model is also derived from a basic formulation of the management problem that integrates reliability and penalty considerations.

3. Figures 23.2 and 23.3 give a comparison of the results for the recourse and the expectation-variance models when we vary β (the budget level). Statistical parameters (expectation, standard deviation and extremes) of the water quality indicators gained from Monte Carlo procedure are illustrated in Figure 23.2 for the Keszthely basin as a function of the available budget β.

In Figure 23.3, we record the changes in the two major control variables (x_{SN1} and x_{D1}) associated to the treatment plant of Zalaegerszeg and the (second) reed lake segment of the Kis-Balaton system (see Figure 23.1). There is a significant trade-off between these two variables. For decision making purposes, it is important to observe that there are four ranges of possible values of β, in which the solution has different characteristics.

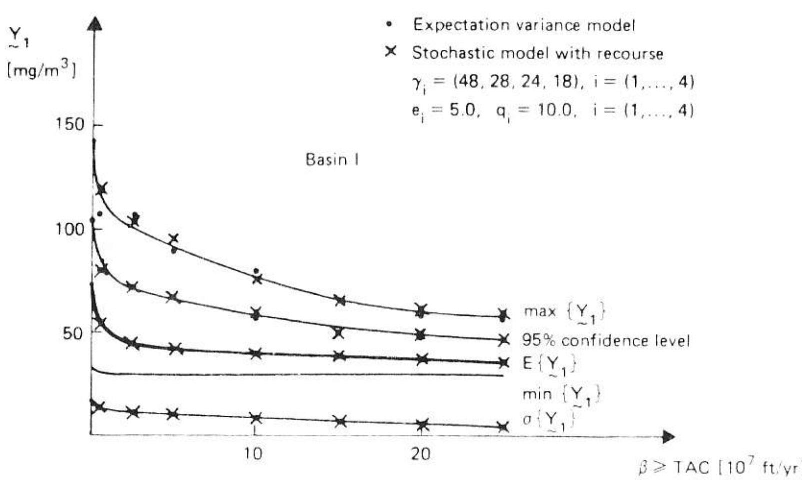

Figure 23.2. Water quality indicator $(\text{Chl} - a)_{\max}$ as a function of the total annual cost.

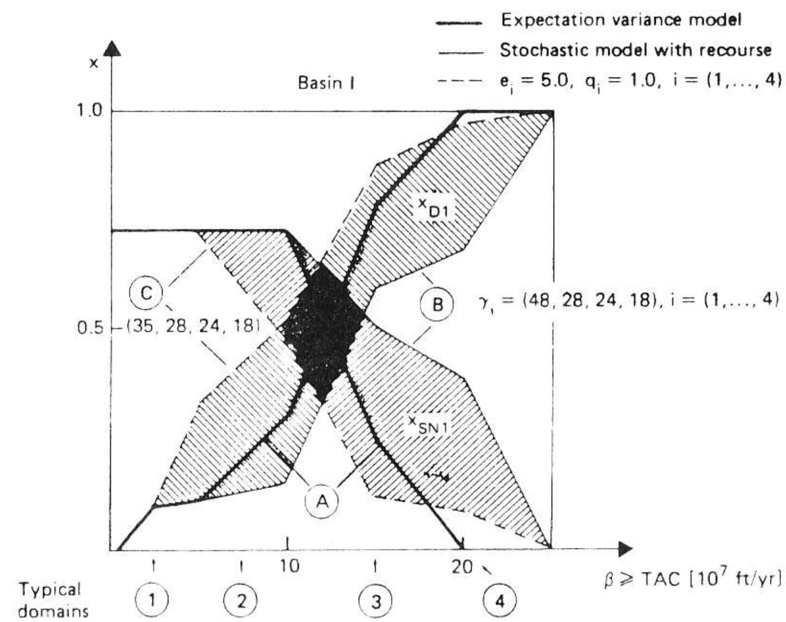

Figure 23.3. Change of major decision variables.

As seen from Figures 23.2-23.3, the two models produce practically the same results in terms of the water quality indicators (including also their distribution). With respect to details there are minor deviations. According to Figure 23.3, the expectation-variance model gives more emphasis to fluctuations in water quality and consequently to reservoir projects, than the stochastic recourse model (see the basic case, B, with the parameters specified), and this is in accordance with the fact that the role of the variance is overstressed in the expectation-variance model.

From this quick comparison of the performances of the two models, we may conclude that the more precise stochastic model validates the use of the expectation-variance model in the case of Lake Balaton.

A more detailed analysis, and further discussion on the role of parameters γ_i, e_i and q_i, and comparison between deterministic models and the stochastic models is given in Section 8 of Somlyódy and Wets [9].

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